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Calculation of load capacity of spur and helical gears — Application for industrial gears

Calcul de la capacité de charge des engrenages à denture droite et hélicoïdale — Application aux engrenages industriels

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 9085 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

Annexes A and B form a normative part of this International Standard. Annexes C and D are for information only.

Introduction

Procedures for the calculation of the load capacity of general spur and helical gears with respect to pitting and bending strength appear in ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5. This International Standard is derived from ISO 6336-1, ISO 6336-2 and ISO 6336-3 by the use of specific methods and assumptions which are considered to be applicable to industrial gears. Its application requires the use of allowable stresses and material requirements which are to be found in ISO 6336-5.

Calculation of load capacity of spur and helical gears — Application for industrial gears

1 Scope

The formulae specified in this International Standard are intended to establish a uniformly acceptable method for calculating the pitting resistance and bending strength capacity of industrial gears with spur or helical teeth.

The rating formulae in this International Standard are not applicable to other types of gear tooth deterioration such as plastic yielding, micropitting, scuffing, case crushing, welding and wear, and are not applicable under vibratory conditions where there may be an unpredictable profile breakdown. The bending strength formulae are applicable to fractures at the tooth fillet, but are not applicable to fractures on the tooth working profile surfaces, failure of the gear rim, or failures of the gear blank through web and hub. This International Standard does not apply to teeth finished by forging or sintering. It is not applicable to gears which have a poor contact pattern.

This International Standard provides a method by which different gear designs can be compared. It is not intended to assure the performance of assembled drive gear systems. Neither is it intended for use by the general engineering public. Instead, it is intended for use by the experienced gear designer who is capable of selecting reasonable values for the factors in these formulae based on knowledge of similar designs and awareness of the effects of the items discussed.

CAUTION — The user is cautioned that the calculated results of this International Standard should be confirmed by experience.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 54:1996, *Cylindrical gears for general and heavy engineering — Modules*

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*[1)]

ISO 4287:1997, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters*

1) This was corrected and reprinted in 1997.

ISO 6336-1:1996, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*

ISO 6336-2:1996, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting)*

ISO 6336-3:1996, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*

ISO 6336-5:1996, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*

ISO 9084:2000, *Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements*

ISO/TR 10495:1997, *Cylindrical gears — Calculation of service life under variable loads — Conditions for cylindrical gears accordance with ISO 6336*

ISO/TR 13593:1999, *Enclosed gear drives for industrial applications*

3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 1122-1 apply. For the symbols, see Table 1.

Table 1 — Symbols and abbreviations used in this International Standard

Symbol	Description or term	Unit
a	centre distance[a]	mm
b	facewidth	mm
b_B	facewidth of an individual helix of a double helical gear	mm
b_H	facewidth (pitting)	mm
b_F	facewidth (tooth root)	mm
b_{red}	reduced facewidth (facewidth minus end reliefs)	mm
b_s	web thickness	mm
$b_{I(II)}$	length of end relief	mm
c_γ	mean value of mesh stiffness per unit facewidth	N/(mm·µm)
c'	maximum tooth stiffness of one pair of teeth per unit facewidth (single stiffness)	N/(mm·µm)
$d_{a1,2}$	tip diameter of pinion (or wheel)	mm
$d_{an1,2}$	tip diameter of pinion (or wheel) of virtual spur gear	mm
$d_{b1,2}$	base diameter of pinion (or wheel)	mm
$d_{bn1,2}$	base diameter of pinion (or wheel) of virtual spur gear	mm
$d_{en1,2}$	diameter of circle through outer point of single pair tooth contact of pinion, wheel of virtual spur gear	mm
$d_{f1,2}$	root diameter of pinion, wheel	mm
$d_{m1,2}$	diameter at mid-tooth depth of pinion, wheel	mm
$d_{n1,2}$	reference diameter of pinion, wheel of virtual spur gear	mm
d_{sh}	nominal shaft diameter for bending	mm
d_{shi}	internal diameter of hollow shaft	mm
$d_{w1,2}$	pitch diameter of pinion, wheel	mm
d_{Nf2}	diameter of a circle near the tooth-roots, containing the limits of the usable flanks of an internal gear or the larger external gear of a mating gear	mm
$d_{1,2}$	reference diameter of pinion, wheel	mm
$f_{f\ eff}$	effective profile form deviation	µm
$f_{f\alpha}$	profile form deviation (the value for the total profile deviation F_α may be used alternatively for this, if tolerances complying with ISO 1328-1 are used)	µm
f_{ma}	helix deviation due to manufacturing inaccuracies	µm
f_{pb}	transverse base pitch deviation (the values of f_{pt} may be used for calculations in accordance with ISO 6336:1996, using tolerances complying with ISO 1328-1)	µm
$f_{pb\ eff}$	effective transverse base pitch deviation	µm
f_{sh}	helix deviation due to elastic deflections	µm
$f_{H\beta}$	tooth alignment deviation (not including helix form deviation)	µm
g_α	path length of contact	mm
h	tooth depth	mm

Table 1 (*continued*)

Symbol	Description or term	Unit
h_a	addendum	mm
h_{a0}	tool addendum	mm
h_{f2}	dedendum of tooth of an internal gear	mm
h_{fP}	dedendum of basic rack of cylindrical gears	mm
h_{Fe}	bending moment arm for load application at the outer point of single pair tooth contact	mm
h_{Nf2}	dedendum of tooth of an internal gear, containing the limits of the usable flanks of an internal gear or the larger external gear of a mating gear	mm
l	bearing span	mm
m_n	normal module	mm
m_{red}	reduced gear pair mass per unit facewidth referenced to the line of action	kg/mm
n_E	resonance speed	min^{-1}
$n_{1,2}$	rotation speed of pinion, wheel	min^{-1}
p_{bn}	normal base pitch	mm
p_{bt}	transverse base pitch	mm
pr	protuberance of the tool	mm
q	finishing stock allowance	mm
q_s	notch parameter $s_{Fn} / 2\rho_F$	—
q_{sT}	notch parameter of standard reference test gear	—
r_b	base radius	mm
s	pinion offset from shaft centre line	mm
s_{Fn}	tooth-root chord at the critical section	mm
s_R	rim thickness	mm
s_{pr}	residual fillet undercut	mm
u	gear ratio $\lvert u \rvert = \lvert z_2/z_1 \rvert \geqslant 1$ [a]	—
v	circumferential speed (without subscript: at reference circle ≈ circumferential speed at working pitch circle)	m/s
$x_{1,2}$	profile shift coefficient of pinion, wheel	—
y_f	running-in allowance (pitch deviation)	µm
y_p	running-in allowance (profile deviation)	µm
y_α	running-in allowance for a gear pair	µm
y_β	running-in allowance (equivalent misalignment)	µm
z_n	virtual number of teeth of a helical gear	—
$z_{1,2}$	number of teeth of pinion, wheel [a]	—
B	total facewidth of a double helical gear including the gap	mm
B_f	running-in parameter for determination of constant K	—
B_k	running-in parameter for determination of constant K	—

Table 1 (*continued*)

Symbol	Description or term	Unit
B_p	running-in parameter for determination of constant K	—
$B_{1,2}$	constants for determination of $F_{\beta x}$	—
B^*	constant for determination of the pinion offset	—
C_a	tip relief	µm
C_{ay}	tip relief resulting from running-in	µm
$C_{v1,2,3}$	constants for determination of constant K	—
C_B	basic rack factor	—
C_R	gear blank factor	—
C_β	crowning height	µm
$C_{1...9}$	constants for determination of q_s	—
E	modulus of elasticity, Young's modulus	N/mm²
E	auxiliary value for calculation of Y_F	—
F_m	mean transverse force at the reference cylinder (= $F_t\, K_A\, K_v$)	N
F_t	(nominal) transverse tangential force at reference cylinder	N
$F_{t\,max}$	maximum transverse tangential force at reference cylinder	N
F_{tH}	determinant transverse force at the reference cylinder (= $F_t\, K_A\, K_v\, K_{H\beta}$)	N
F_β	total helix deviation	µm
$F_{\beta x}$	initial equivalent misalignment (before running-in)	µm
G	auxiliary value for calculation of Y_F	—
H	auxiliary value for calculation of Y_F	—
$J^*_{1,2}$	polar moment of inertia per unit face width	Kg/mm
K	constant for determination of K_v	—
K_v	dynamic factor	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (root stress)	—
$K_{F\beta}$	face load factor (root stress)	—
$K_{H\alpha}$	transverse load factor (contact stress)	—
$K_{H\beta}$	face load factor (contact stress)	—
K_γ	mesh load factor (takes into account the uneven distribution of the load between meshes for multiple transmission paths)	—
$K_{1,2}$	constant	—
K'	constant for the pinion offset in relation to the torqued end	—
L	tooth root chord at the critical section, related to the bending moment arm relevant to load application at the outer point of single pair tooth contact	—
N	resonance ratio	—
N_F	exponent	—

Table 1 *(continued)*

Symbol	Description or term	Unit
N_L	number of load cycles	—
N_S	resonance ratio in the main resonance range	—
$M_{1,2}$	auxiliary values for the determination of $Z_{B,D}$	—
P	transmitted power	kW
P_{max}	maximum transmitted power	kW
Ra	arithmetic mean roughness value (as specified in ISO 4287:1997)	µm
Rz	mean peak-to-valley roughness (as specified in ISO 4287:1997)	µm
Rz_{10}	mean peak-to-valley roughness for the gear pair	µm
S_F	safety factor from tooth breakage	—
$S_{F\,min}$	minimum safety factor (tooth breakage)	—
S_H	safety factor from pitting	—
$S_{H\,min}$	minimum safety factor (pitting)	—
$T_{1,2}$	pinion torque (nominal); wheel torque	Nm
T_{max}	maximum torque	Nm
Y_F	tooth form factor	—
Y_N	life factor for tooth-root stress	—
Y_{NT}	life factor for tooth-root stress for reference test conditions	—
$Y_{R\,rel\,T}$	surface factor	—
Y_S	stress correction factor	—
Y_X	size factor (tooth root)	—
Y_β	helix angle factor (tooth root)	—
$Y_{\delta\,rel\,T}$	relative notch sensitivity factor	—
Y_ε	contact ratio factor (tooth root)	—
Z_v	speed factor	—
$Z_{B,D}$	single pair tooth contact factors for the pinion, wheel	—
Z_E	elasticity factor	$\sqrt{N/mm^2}$
Z_H	zone factor	—
Z_L	lubricant factor	—
Z_N	life factor for contact stress	—
Z_{NT}	life factor for contact stress for reference test conditions	—
Z_R	roughness factor affecting surface durability	—
Z_W	work-hardening factor	—
Z_X	size factor (pitting)	—
Z_β	helix angle factor (pitting)	—
Z_ε	contact ratio factor (pitting)	—

Table 1 (*continued*)

Symbol	Description or term	Unit
α_{en}	pressure angle at the outer point of single pair tooth contact of virtual spur gears	°
α_n	normal pressure angle	°
α_t	transverse pressure angle	°
α_{wt}	transverse pressure angle at the pitch cylinder	°
α_{Fen}	load direction angle, relevant to direction of application of load at the outer single pair tooth contact of virtual spur gears	°
α_{Pn}	normal pressure angle of the basic rack for cylindrical gears	°
β	helix angle at the reference cylinder	°
β_b	base helix angle	°
γ_e	auxiliary angle for determination of α_{Fen}	°
δ_{bth}	combined deflection of mating teeth assuming even load distribution over the facewidth	µm
ε_α	transverse contact ratio	—
$\varepsilon_{\alpha n}$	transverse contact ratio of a virtual spur gear	—
ε_β	axial overlap ratio	—
ε_γ	total contact ratio ($\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$)	—
ν	Poisson's contact ratio	—
θ	auxiliary value for calculation of Y_F	—
ρ_{a0}	tip radius of the tool	mm
ρ_{fP}	root fillet radius of the basic rack for cylindrical gears	mm
ρ_{rel}	radius of relative curvature	mm
ρ_F	tooth-root fillet radius at the critical section	mm
ρ'	slip-layer thickness	mm
σ_B	tensile strength	N/mm^2
σ_F	tooth-root stress	N/mm^2
$\sigma_{F\,lim}$	nominal stress number (bending)	N/mm^2
σ_{FE}	allowable stress number (bending) = $\sigma_{F\,lim}\,Y_{ST}$	N/mm^2
σ_{FG}	tooth-root stress limit	N/mm^2
σ_{FP}	permissible tooth-root stress	N/mm^2
σ_{F0}	nominal tooth-root stress	N/mm^2
σ_H	calculated contact stress	N/mm^2
$\sigma_{H\,lim}$	allowable stress number (contact)	N/mm^2
σ_{HG}	modified allowable stress number = $\sigma_{HP}\,S_{H\,min}$	N/mm^2
σ_{HP}	permissible contact stress	N/mm^2
σ_{H0}	nominal contact stress	N/mm^2

Table 1 (*continued*)

Symbol	Description or term	Unit
σ_S	yield point	N/mm2
$\sigma_{0,2}$	0,2 % proof stress	N/mm2
χ^*	relative stress gradient in the root of a notch	mm-1
χ^*_P	relative stress gradient in a smooth polished test piece	mm-1
χ^*_T	relative stress gradient in the root of the standard reference test gear	mm-1
$\omega_{1,2}$	angular velocity of pinion, wheel	rad/s
[a] For external gear pairs, a, u, z_1 and z_2 are positive; for internal gear pairs, a, u and z_2 are negative, and z_1 positive.		

4 Application

4.1 Design, specific applications

4.1.1 General

Gear designers must recognize that requirements for different applications vary considerably. Use of the procedures of this International Standard for specific applications demands a careful appraisal of all applicable considerations, in particular:

— the allowable stress of the material and the number of load repetitions;

— the consequences of any percentage of failure (failure rate);

— the appropriate factor of safety.

Design considerations to prevent fractures emanating from stress raisers in the tooth flank, tip chipping and failures of the gear blank through the web or hub should be analysed by general machine design methods.

Any variances according to the following shall be reported in the calculation statement.

a) If a more refined method of calculation is desired or if compliance with the restrictions given in 4.1 is for any reason impractical, relevant factors may be evaluated according to the basic standard or another application standard.

b) Factors derived from reliable experience or test data may be used instead of individual factors according to this International Standard. Concerning this, the criteria for Method A in 4.1.8.1 of ISO 6336-1:1996 are applicable.

In other respects, rating calculations shall be strictly in accordance with this International Standard wherever stresses, safety factors etc. are to be classified as being in accordance with this International Standard.

This International Standard recognizes the following types of industrial drive design.

— **Catalogue enclosed drives** are designed to nominal load ratings for sale from catalogues or from stock. The actual loads and operation conditions are not exactly known at the time of design.

NOTE The actual loads for each application are evaluated to select an appropriately sized unit from the catalogue. A selection factor, based on experience with similar applications, is often used to reduce the catalogue rating to match the application conditions (see ISO TR 13593).

— **Custom designed drives** are aimed at a specific application where the operating conditions are known or specified at the time of design.

This International Standard is applicable when the wheel blank, shaft/hub connections, shafts, bearings, housings, threaded connections, foundations and couplings conform to the requirements regarding accuracy, load capacity and stiffness which form the basis for the calculation of the load capacity of gears.

Although the method described in this International Standard is mainly intended for recalculation purposes, by means of iteration it can also be used to determine the load capacities of gears. The iteration is accomplished by selecting a load and calculating the corresponding safety factor against pitting, S_{H1}, for the pinion. If S_{H1} is greater than $S_{H\,min}$, the load is increased; if it is smaller than $S_{H\,min}$, the load is reduced. This is done until the chosen load corresponds to $S_{H1} = S_{H\,min}$. The same method is used for the wheel ($S_{H2} = S_{H\,min}$), and also for the safety factors against tooth breakage, $S_{F1} = S_{F2} = S_{F\,min}$.

4.1.2 Gear data

This International Standard is applicable within the following constraints.

a) Types of gear:

— external and internal, involute spur, helical and double helical gears;

— for double helical gears, it is assumed that the total tangential load is evenly distributed between the two helices; if this is not the case (e.g. due to externally applied axial forces), this shall be taken into account; the two helices are treated as two single helical gears in parallel.

b) Range of speeds:

— n_1 less than or equal to 3 600 min^{-1} (synchronous speed of two-pole motor at 60 Hz current frequency)[2];

— subcritical range of speed (see K_v in 5.6);

— at speeds of $v < 1$ m/s, gear load capacity is often limited by wear.

c) Gear accuracy:

— accuracy grade 10 or better according to ISO 1328-1 (affects K_v, $K_{H\alpha}$ and $K_{H\beta}$).

d) Range of the transverse contact ratios of virtual spur gear pairs:

— $1{,}2 < \varepsilon_\alpha < 1{,}9$ (affects c', c_γ, K_v, $K_{H\beta}$, $K_{F\alpha}$, $K_{H\alpha}$ and $K_{F\beta}$).

e) Range of helix angles:

— β less than or equal to 30° (affects c', c_γ, K_v and $K_{H\beta}$).

4.1.3 Pinion and pinion shaft

This International Standard is applicable to pinions integral with shafts or bored pinions with $s_R/d_1 \geqslant 0{,}2$ (this affects c', c_γ, K_v, $K_{H\beta}$). It is assumed that the bored pinions will be mounted on solid shafts or on hollow shafts with $d_{shi}/d_{sh} < 0{,}5$ (this affects $K_{H\beta}$).

4.1.4 Wheel blank, wheel rim

The given formulae are valid for spur and helical gears with a minimum rim thickness under the root of $s_R \geqslant 3{,}5\, m_n$. The calculation of $K_{H\beta}$ assumes that wheel and wheel shaft are sufficiently stiff such that their deflections can be ignored.

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2) For higher speeds, the requirements of ISO 6336 or ISO 9084 apply.

4.1.5 Materials

These include steels, nodular cast iron and grey cast iron (this affects Z_E, $\sigma_{H\,lim}$, σ_{FE}, K_v, $K_{H\beta}$, $K_{F\beta}$, $K_{H\alpha}$ and $K_{F\alpha}$). For materials and their abbreviations used in this International Standard, see Table 2.

Table 2 — Materials

Material	Abbreviation
Steel ($\sigma_B < 800$ N/mm^2)	St
Cast steel, alloy or carbon, ($\sigma_B \geqslant 800$ N/mm^2)	St (cast)
Through-hardening steel, alloy or carbon, through hardened ($\sigma_B \geqslant 800$ N/mm^2)	V
Grey cast iron	GG
Nodular cast iron (pearlitic, bainitic, ferritic structure)	GGG (perl., bai., ferr.)
Black malleable cast iron (pearlitic structure)	GTS (perl.)
Case-hardened steel, case hardened	Eh
Steel and GGG, flame or induction hardened	IF
Nitriding steel, nitrided	NT (nitr.)
Through-hardening and case-hardening steel, nitrided	NV (nitr.)
Through-hardening and case-hardening steel, nitrocarburized	NV (nitrocar.)

4.1.6 Lubrication

The calculation procedures are valid for oil lubricated gears having sufficient lubricant of suitable viscosity at the gear mesh and when the working temperature is also suitable (this affects lubricant film formation, i.e. the factors Z_L, Z_v and Z_R).

4.2 Safety factors

It is necessary to distinguish between the safety factor relative to pitting, S_H, and the safety factor relative to tooth breakage, S_F.

For a given application, adequate gear load capacity is demonstrated by the computed values of S_H and S_F being equal to or greater than the values $S_{H\,min}$ and $S_{F\,min}$, respectively.

Choice of the value of a safety factor should be based on the degree of confidence in the reliability of the available data and the consequences of possible failures.

Important factors to be considered are

a) that the validity of the material values in ISO 6336-5 is for 1 % probability of damage,

b) the specified quality and the effectiveness of quality control at all stages of manufacture,

c) the accuracy of specification of the service duty and external conditions, and

d) that tooth breakage is often considered to be a greater hazard than pitting.

Therefore, the chosen value for $S_{F\,min}$ should be greater than the value chosen for $S_{H\,min}$.

For calculation of the actual safety factor, see 6.1.5 (S_H, pitting) and 7.1.4 (S_F, tooth breakage). For minimum safety factors see 6.12 (pitting) and 7.9 (tooth breakage). However, it is recommended that the minimum values of the safety factors should be agreed upon between the purchaser and the manufacturer.

4.3 Input data

The following data shall be available for the calculations:

a) gear data:

a, z_1, z_2, m_n, d_1, d_{a1}, d_{a2}, b, b_H, b_F, x_1, x_2, α_n, β, ε_α, ε_β (see ISO 53, ISO 54) (see 4.4 for definition of b, b_H and b_F face widths);

b) cutter basic rack tooth profile:

h_{a0}, ρ_{a0};

c) design and manufacturing data:

C_{a1}, C_{a2}, f_{pb}, $S_{H\,min}$, $S_{F\,min}$, Ra_1, Ra_2, Rz_1, Rz_2;

materials, material hardnesses and heat treatment details; gear accuracy grades, bearing span l, positions of gears relative to bearings; dimensions of pinion shaft d_{sh} and, when applicable, helix modification (crowning, end relief);

d) power data:

P or T or F_t, n_1, v_1, details of driving and driven machines.

Requisite geometrical data can be calculated according to national standards.

Information to be exchanged between manufacturer and purchaser should include data specifying material preferences, lubrication, safety factor and externally applied forces due to vibrations and overloads (application factor).

4.4 Face widths

The following face widths have to be distinguished.

— b: the smaller of the facewidths of pinion and wheel measured at the pitch circles (for a double helical gear $b_H = 2\,b_B$). Chamfers or rounding of tooth ends are to be ignored. Where the facewidths are offset, the length of the face in contact shall be used.

— b_H: the facewidth at the pitch cylinder of the gear (for a double helical gear $b_H = 2\,b_B$). When the facewidth b_H is larger than that of its mating gear, b_H shall be based on the smaller facewidth, ignoring any intentional transverse chamfers or tooth-end rounding. Neither unhardened portions of surface-hardened gear tooth flanks nor the transition zones shall be included. Where the facewidths are offset, the length of the face in contact shall be used.

— b_F: the facewidth at the root cylinder of the gear (for a double helical gear $b_F = 2\,b_B$). When the facewidth b_F is larger than that of its mating gear, b_F shall be based on the smaller facewidth plus a length, not exceeding one module of any extension at each end. However, if it is foreseen that because of crowning or because end relief contact does not extend to the end of face, then the smaller facewidth shall be used for both pinion and wheel. Where the facewidths are offset, the length of the face in contact shall be used.

4.5 Numerical equations

The units listed in clause 3 shall be used in all calculations. Information which will facilitate the use of this International Standard is provided in annex C of ISO 6336-1:1996.

5 Influence factors

5.1 General

The influence factors K_v, $K_{H\alpha}$, $K_{H\beta}$, $K_{F\alpha}$ and $K_{F\beta}$ are all dependent on the tooth load. Initially this is the applied load (nominal tangential load multiplied by the application factor).

The factors are also interdependent and shall therefore be calculated successively as follows:

a) K_v with the applied tangential load $F_t K_A$;

b) $K_{H\beta}$ or $K_{F\beta}$ with the recalculated load $F_t K_A K_v$;

c) $K_{H\alpha}$ or $K_{F\alpha}$ with the applied tangential load $F_t K_A$.

When a gear drives two or more mating gears, it is necessary to substitute K_A by $K_A K_\gamma$. If possible, the mesh load factor, K_γ, should be determined by measurement; alternatively, its value may be estimated from the literature.

5.2 Nominal tangential load, F_t, nominal torque, T, nominal power, P

The nominal tangential load, F_t, is determined in the transverse plane at the reference cylinder. It is based on the input torque to the driven machine. This is the torque corresponding to the heaviest regular working condition. Alternatively, the nominal torque of the prime mover can be used as a basis if it corresponds to the torque requirement of the driven machine, or some other suitable basis can be chosen.

$$F_t = \frac{2\,000\,T_{1,2}}{d_{1,2}} = \frac{19\,098 \times 1\,000\,P}{d_{1,2}\,n_{1,2}} = \frac{1\,000\,P}{v} \tag{1}$$

$$T_{1,2} = \frac{F_t\,d_{1,2}}{2\,000} = \frac{1\,000\,P}{\omega_{1,2}} = \frac{9\,549\,P}{n_{1,2}} \tag{2}$$

$$P = \frac{F_t\,v}{1\,000} = \frac{T_{1,2}\,\omega_{1,2}}{1\,000} = \frac{T_{1,2}\,n_{1,2}}{9\,549} \tag{3}$$

$$v = \frac{d_{1,2}\,\omega_{1,2}}{2\,000} = \frac{d_{1,2}\,n_{1,2}}{19\,098} \tag{4}$$

$$\omega_{1,2} = \frac{\pi n_{1,2}}{30} = \frac{2\,000\,v}{d_{1,2}} = \frac{n_{1,2}}{9\,549} \tag{5}$$

5.3 Non-uniform load, non-uniform torque, non-uniform power

When the transmitted load is not uniform, consideration should be given not only to the peak load and its anticipated number of cycles, but also to intermediate loads and their numbers of cycles. This type of load is classed as a *duty cycle* and may be represented by a load spectrum. In such cases, the cumulative fatigue effect of the duty cycle is considered in rating the gear set. A method of calculating the effect of the loads under this condition is given in ISO TR 10495.

5.4 Maximum tangential load, $F_{t\,max}$, maximum torque, T_{max}, maximum power, P_{max}

This is the maximum tangential load $F_{t\,max}$, (or corresponding torque, T_{max}, corresponding power, P_{max}) in the variable duty range. Its magnitude can be limited by a suitably responsive safety clutch. $F_{t\,max}$, T_{max} and P_{max} shall be known when safety from pitting damage and from sudden tooth breakage due to loading corresponding to the static stress limit is determined (see 5.5).

5.5 Application factor, K_A

5.5.1 General

The factor K_A adjusts the nominal load F_t, in order to compensate for incremental gear loads from external sources. These additional forces are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of the system, including shafts and couplings used in service.

It is recommended that the purchaser and manufacturer/designer agree on the value of the application factor.

5.5.2 Method A — Factor $K_{A\text{-}A}$

K_A is determined in this method by means of careful measurements and a comprehensive analysis of the system, or on the basis of reliable operational experience in the field of application concerned (see 5.3).

5.5.3 Method B — Factor $K_{A\text{-}B}$

If no reliable data, obtained as described in 5.5.2, is available, or even as early as the first design phase, it is possible to use the guideline values for K_A as described in annex C.

5.6 Internal Dynamic Factor, K_v

5.6.1 General

The dynamic factor relates the total tooth load, including internal dynamic effects of a "multi-resonance" system, to the transmitted tangential tooth load.

Method B of ISO 6336-1:1996 with modifications is used in this International Standard. When agreed between manufacturer and purchaser, or when determining the catalogue presentation of the capacities of catalogue enclosed drives, Method E of ISO 6336-1:1996 may be used to estimate the dynamic factor.

In this procedure it is assumed that the gear pair consists of an elementary single mass and spring system comprising the equivalent masses of pinion and wheel, and the mesh stiffness of the contacting teeth. It is also assumed that each gear pair functions as a single stage pair, i.e. the influence of other stages in a multiple-stage gear system is ignored. This assumption is only tenable when the torsional stiffness (measured at the base radius of the gears) of the shaft common to a wheel and a pinion is less than the mesh stiffness. See 5.6.3 and annex A for the procedure dealing with very stiff shafts.

Forces caused by torsional vibrations of the shafts and coupled masses are not covered by K_v. These forces should be included with other externally applied forces (e.g. with the application factor).

In multiple mesh gear trains, there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh. When such gears run in the supercritical range, analysis by Method A is recommended. See ISO 6336-1:1996, 6.3.1.

The specific load for the calculation of K_v is $(F_t K_A)/b$.

If $(F_t K_A)/b > 100$ N/mm, then $F_m/b = (F_t K_A)/b$,

if $(F_t K_A)/b \leqslant 100$ N/mm, then $F_m/b = 100$ N/mm.

When the specific loading $(F_t K_A)/b < 50$ N/mm, a particular risk of vibration exists (under some circumstances, with separation of working tooth flanks), above all for spur or helical gears of coarse accuracy grade running at higher speed.

5.6.2 Calculation of the parameters required for evaluation of K_v

5.6.2.1 Calculation of the equivalent mass, m_{red}

a) Calculation of the equivalent mass m_{red} of a single-stage gear pair

$$m_{red} = \frac{J_1^* J_2^*}{J_1^* r_{b2}^2 + J_2^* r_{b1}^2} \tag{6}$$

where

m_{red} is the equivalent mass of a gear pair, i.e. of the mass per unit facewidth of each gear, referred to its base radius or to the line of action;

$J^*_{1,2}$ are the polar moments of inertia per unit facewidth;

$r_{b1,2}$ are the base radii (= 0,5 $d_{b1,2}$).

b) Calculation of equivalent mass, m_{red}, of a multi-stage gear pair

See annex A.

c) Calculation of equivalent mass, m_{red}, of gears of less common designs

For information on the following cases, see A.1.2:

— pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter;

— two rigidly connected, coaxial gears;

— one large wheel driven by two pinions;

— planetary gears;

— idler gears.

5.6.2.2 Determination of the resonance running speed (main resonance) of a gear pair

a) Resonance running speed, n_{E1}, of the pinion, in reciprocal minutes:

$$n_{E1} = \frac{30 \times 10^3}{\pi z_1} \sqrt{\frac{c_\gamma}{m_{red}}} \text{ min}^{-1} \tag{7}$$

with c_γ from annex B.

b) Resonance ratio, N

The ratio of pinion speed to resonance speed, the resonance ratio, N, is determined as follows.

$$N = \frac{n_1}{n_{E1}} = \frac{n_1 \pi z_1}{30\,000} \sqrt{\frac{m_{red}}{c_\gamma}} \tag{8}$$

The resonance running speed may be above or below the running speed calculated from equation (8) because of stiffnesses which have not been included (e.g. the stiffnesses of shafts, bearings or housings) and as a result of damping. For reasons of safety, the resonance range is defined by the following.

$$N_S < N \leqslant 1{,}15 \tag{9}$$

At loads such that $(F_t K_A)/b$ is less than 100 N/mm, the lower limit of resonance ratio N_S is determined as follows:

— if $(F_t K_A)/b < 100$ N/mm, then

$$N_S = 0{,}5 + 0{,}35 \sqrt{\frac{F_t K_A}{b \times 100}} \tag{10}$$

— if $(F_t K_A)/b \geqslant 100$ N/mm, then

$$N = 0{,}85 \tag{11}$$

5.6.2.3 Gear accuracy and running-in parameters B_p, B_f, B_k

B_p, B_f and B_k are non-dimensional parameters used to take into account the effect of tooth deviations and profile modifications on the dynamic load. [3)]

$$B_p = \frac{c' f_{pb\,eff}}{(F_t K_A)/b} \tag{12}$$

$$B_f = \frac{c' f_{f\,eff}}{(F_t K_A)/b} \tag{13}$$

$$B_k = \left| 1 - \frac{c' C_a}{(F_t K_A)/b} \right| \tag{14}$$

with

c' from annex B;

C_a design amount for profile modification (tip relief at the beginning and end of tooth engagement). A value C_{ay} from running-in shall be substituted for C_a in equation (14) in the case of gears without a specified profile modification. The value of C_{ay} can be obtained from Table 3.

The effective base pitch and profile deviations are those which are present after running-in. The values of $f_{pb\,eff}$ and $f_{f\,eff}$ are determined by deducting estimated running-in allowances y_p and y_f as follows:

$$f_{pb\,eff} = f_{pb1} - y_{p1} \quad \text{or} \quad f_{pb\,eff} = f_{pb2} - y_{p2} \tag{15}$$

whichever is the greater;

---

3) The amount C_a of tip relief may only be allowed for gears of accuracy grades in the range 0 to 6 as specified in ISO 1328-1:1995.

$$f_{\text{f eff}} = f_{\text{f}\alpha 1} - y_{\text{f}1} \quad \text{or} \quad f_{\text{f eff}} = f_{\text{f}\alpha 2} - y_{\text{f}2} \tag{16}$$

whichever is the greater.

5.6.2.4 Running-in allowance, y_α

a) For St, St (cast), V, GGG (perl., bai.), GTS (perl.) [4]

$$y_{\text{p}} = y_\alpha = \frac{160}{\sigma_{\text{H lim}}} f_{\text{pb}} \tag{17}$$

$$y_{\text{f}} = \frac{160}{\sigma_{\text{H lim}}} f_{\text{f}\alpha} \tag{18}$$

b) For GG, GGG (ferr.) [4]

$$y_{\text{p}} = y_\alpha = 0{,}275\, f_{\text{pb}} \tag{19}$$

$$y_{\text{f}} = 0{,}275\, f_{\text{f}\alpha} \tag{20}$$

c) For Eh, IF, NT (nitr.), NV (nitr.), NV (nitrocar.) [4]

$$y_{\text{p}} = y_\alpha = 0{,}075\, f_{\text{pb}} \tag{21}$$

$$y_{\text{f}} = 0{,}075\, f_{\text{f}\alpha} \tag{22}$$

5.6.3 Dynamic factor in the subcritical range ($N \leqslant N_{\text{S}}$)

In this sector, resonances may exist if the tooth mesh frequency coincides with $N = 1/2$ and $N = 1/3$. The risk of this is slight in the case of precision helical or spur gears, if the latter have suitable profile modification (gears as specified in ISO 1328-1:1995, accuracy grade 6 or better).

When the contact ratio of straight spur gears is small or if the quality is of low grade, K_{v} can be just as great as in the main resonance-speed range. If this occurs, the design or operating parameters should be altered.

Resonances at $N = 1/4$, 1/5, etc. are seldom troublesome because the associated vibration amplitudes are usually small.

For gear pairs where the stiffnesses of the driving and driven shafts are not equal, in the range $N \approx 0{,}2$ to 0,5, the tooth contact frequency can excite natural frequencies when the torsional stiffness c of the stiffer shaft, referred to the line of action, is of the same order of magnitude as the tooth stiffness, i.e. if c/r_{b}^2 is of the order of magnitude of c_γ. When this is so, dynamic load increments can exceed values calculated using equation (23).

$$K_{\text{v}} = (NK) + 1 \tag{23}$$

$$K = \left(C_{\text{v}1} B_{\text{p}}\right) + \left(C_{\text{v}2} B_{\text{f}}\right) + \left(C_{\text{v}3} B_{\text{k}}\right) \tag{24}$$

where $C_{\text{v}1}$ and $C_{\text{v}2}$ allow for pitch and profile deviations, while $C_{\text{v}3}$ allows for the cyclic variation of mesh stiffness. See Table 3.

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4) See Table 2 for an explanation of the abbreviations used.

Table 3 — Equations for the calculation of factors C_{V1} to C_{V3} and C_{ay}

	$1 < \varepsilon_\gamma \leqslant 2$	$\varepsilon_\gamma > 2$
C_{V1}	0,32	0,32
C_{V2}	0,34	$\dfrac{0{,}57}{\varepsilon_\gamma \; 0{,}3}$
C_{V3}	0,23	$\dfrac{0{,}096}{\varepsilon_\gamma - 1{,}56}$

$$C_{ay} = \frac{1}{18}\left(\frac{\sigma_{Hlim}}{97} - 18{,}45\right)^2 + 1{,}5$$

NOTE When the material of the pinion (1) is different from that of the wheel (2), C_{ay1} and C_{ay2} are calculated separately: then $C_{ay} = 0{,}5\,(C_{ay1} + C_{ay2})$.

A value C_{ay} resulting from running-in shall be substituted for C_a in equation (14) in the case of gears without a specified profile modification. The value of C_{ay} is obtained from Table 3. See annex B for single tooth stiffness c'.

5.7 Face load factor, $K_{H\beta}$

5.7.1 General

The face load factor adjusts gear tooth stresses, to allow for the effects of uneven load distribution over the facewidth.

Method C2 of ISO 6336-1:1996 with modifications is used in this International Standard, arranged so that account is taken of the influences on mesh alignment, of elastic deformations of the pinion and of manufacturing inaccuracies.

$K_{H\beta}$ shall be calculated from the total mesh misalignment after running-in, $F_{\beta y}$, which comprises the following two components.

— **Systematic error** is taken into account by f_{sh} (mesh misalignment due to shaft deflection) and is primarily caused by pinion shaft deflection, but in principle may include all mechanical deflections which can be evaluated accurately enough in both amount and direction.

— **Random error** is represented by f_{ma} (mesh misalignment due to manufacturing tolerance). The actual direction and amount of misalignment due to manufacturing cannot be evaluated, only the range is limited by manufacturing tolerance (referring to gear accuracy grade).

Application of helix correction and crowning consists of the following.

— **Helix correction** is a lead modification which is applied to compensate for the systematic error. Theoretically, it is possible to apply a helix correction which exactly matches the calculated deflection for a specific load and so eliminates the f_{sh} contribution to $K_{H\beta}$ for that particular load. In practice, however, varying loads and errors in the evaluation of f_{sh} leave a remaining influence on $K_{H\beta}$ which has to be taken into account.

— **Crowning** is a lead modification which comprises the best defensive strategy against the random component of misalignment. Since f_{ma} can be in either direction, crowning should be symmetric to the middle of the face width.

A more exact and comprehensive analysis in accordance with ISO 6336-1 is recommended if the design does not match the requirements of 7.2.3.1 of ISO 6336-1:1996, or if any of the following items have significant influence on mesh alignment:

— elastic deformations not caused by gear mesh forces but by external loads (e.g. belts, chains, couplings);

— elastic deformations of wheel and wheel shaft;

— elastic deformations and manufacturing inaccuracies of the gear case;

— bearing clearances and deflections;

— arrangements different from those shown in Figure 2;

— any manufacturing or other deformations which indicate a need for a more detailed analysis.

When, by this method, a value of $K_{H\beta}$ greater than 2,0 is calculated, the true value will usually be less than this. However, if the calculated value of $K_{H\beta}$ is greater than 1,5, the design should be reconsidered (e.g. shaft stiffness increased, bearing positions changed, helix accuracy improved).

5.7.2 Calculation of $K_{H\beta}$

The specific load for the calculation of $K_{H\beta}$ is $(F_t K_A K_v)/b$.

If $(F_t K_A K_v)/b > 100$ N/mm, then $F_m/b = (F_t K_A K_v)/b$,

if $(F_t K_A K_v)/b \leqslant 100$ N/mm, then $F_m/b = 100$ N/mm.

$$K_{H\beta} = 1 + \frac{F_{\beta y}\, c_\gamma}{2\, F_m / b} \tag{25}$$

applies when $K_{H\beta} \leqslant 2$, with c_γ from annex B.

If $K_{H\beta} > 2$, this International Standard is not applicable.

5.7.3 Mesh misalignment after running-in, $F_{\beta y}$

$$F_{\beta y} = F_{\beta x} - y_\beta \tag{26}$$

where

$F_{\beta x}$ is the mesh misalignment before running-in (see 5.7.4);

y_β is the running-in allowance (see 5.7.8).

5.7.4 Mesh misalignment before running-in, $F_{\beta x}$

5.7.4.1 General

$F_{\beta x}$ is the absolute value of the sum of manufacturing deviations and pinion and shaft deflections, measured in the plane of action.

5.7.4.2 **Custom designed gears** (see clause 4)

a) For gear pairs without verification of the favourable position of the contact pattern[5]:

$$F_{\beta x} = 1{,}33 B_1 f_{sh} + B_2 f_{ma} \tag{27}$$

with B_1 and B_2 taken from Table 4.

b) For gear pairs with verification of the favourable position of the contact pattern (e.g. by adjustment of bearings)[5]:

$$F_{\beta x} = \left| 1{,}33 B_1 f_{sh} - f_{H\beta 5} \right| \tag{28}$$

where

$f_{H\beta 5}$ is the maximum helix slope deviation for ISO accuracy grade 5 (see ISO 1328-1:1995).

By subtracting $f_{H\beta 5}$, allowance is made for the compensatory roles of elastic deformation and manufacturing deviations.

5.7.4.3 **Catalogue enclosed drives** (see clause 4)

For catalogue gear pairs with helix corrections and crowning appropriate for the corresponding catalogue rating or without helix modification, use equation (27). In this case, the location of the gear pair, shaft deflections, bearings, overhung loads, etc. shall be taken into account [6].

Table 4 — Constants for use in equation (27)

No.	Helix modification		Equation constant	
	Type	Amount	B_1	B_2
1	None	—	1	1
2	Central crowning only	$C_\beta = 0{,}5 f_{ma}$[a]	1	0,5
3	Central crowning only	$C_\beta = 0{,}5\,(f_{ma} + f_{sh})$[a]	0,5	0,5
4[b]	Helix correction only	Corrected shape calculated to match torque being analysed	0,1[c]	1,0
5	Helix correction plus central crowning	Case 2 plus case 4	0,1[c]	0,5
6	End relief	appropriate amount $C_{I(II)}$ see annex D	0,7	0,7

[a] Appropriate crowning, C_β, see annex D.

[b] Predominantly applied for applications with constant load conditions.

[c] Valid for very best practice of manufacturing, otherwise higher values appropriate.

5) With a favourable position of the contact pattern, the elastic deformations and the manufacturing deviations compensate each other (see Figure 1, compensatory roles).

6) For example, a gear unit is catalogued at 400 kW when using a selection factor of 1,0. Helix corrections and crown are applied to the gears as per Table 4. There is no verification of favourable contact pattern. The actual power that will be transmitted at catalogue speed will be less than 400 kW. For the 400 kW condition, $F_{\beta x}$ can be calculated per equation (27). When actual transmitted power is lower than 400 kW, resultant tooth stresses will also be lower, although $F_{\beta x}$ and $K_{H\beta}$ will be higher. If the gears in the above unit are also used in other gear units, a nominal amount of crown can be applied to the gear teeth. This amount of crown is selected to fit all possible locations but is not the optimum crown for each power level and location. For these conditions, equation (29) may be used.

Otherwise, for catalogue gear pairs with appropriate helix correction and crowning:

$$F_{\beta x} = \frac{b}{d_1}\left(1{,}75 + \frac{b}{l}\right) f_{ma} \tag{29}$$

When using equation (29), the running-in allowance y_β equals zero.

5.7.5 Minimum values for $K_{H\beta}$

For gear pairs without helix correction or crowning, the minimum value for $K_{H\beta}$ is 1,25 for lowest speed stages (also for single reduction gear drives) and 1,45 for all other stages.

For catalogue enclosed drives with appropriate helix correction and crowning, the minimum value for $K_{H\beta}$ is 1,10 for the lowest speed stage (also for single reduction drives) and 1,25 for all other stages. For custom designed drives with appropriate helix correction and crowning, the minimum value for $K_{H\beta}$ is 1,0.

The minimum value of $K_{H\beta}$ defined above applies at all loads, including overloads.

5.7.6 Equivalent misalignment, f_{sh}

For spur and single helical gears:

$$f_{sh} = \frac{F_m}{b} 0{,}023 \left[\left| 1 + K' \frac{l\,s}{d_1^2} \left(\frac{d_1}{d_{sh}}\right)^4 - 0{,}3 \right| + 0{,}3 \right] \left(\frac{b}{d_1}\right)^2 \tag{30}$$

For double helical gears, the calculation of f_{sh} is relative to the helix nearest to the shaft end which is driven or which drives the load:

$$f_{sh} = \frac{F_m}{b} 0{,}046 \left[\left| 1{,}5 + K' \frac{l\,s}{d_1^2} \left(\frac{d_1}{d_{sh}}\right)^4 - 0{,}3 \right| + 0{,}3 \right] \left(\frac{b_B}{d_1}\right)^2 \tag{31}$$

where

b $= 2\, b_B$,

b_B is the width of one helix.

In equations (30) and (31), K', s and l are according to Figure 2.

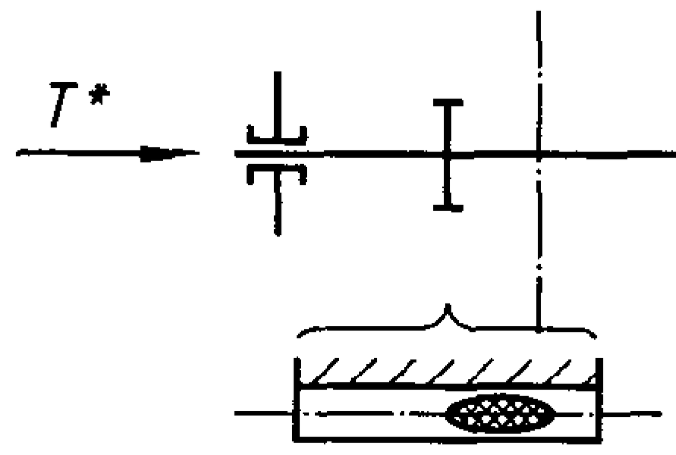
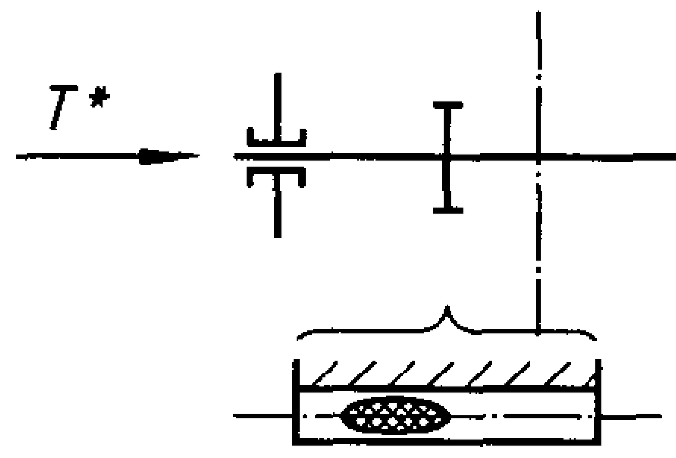
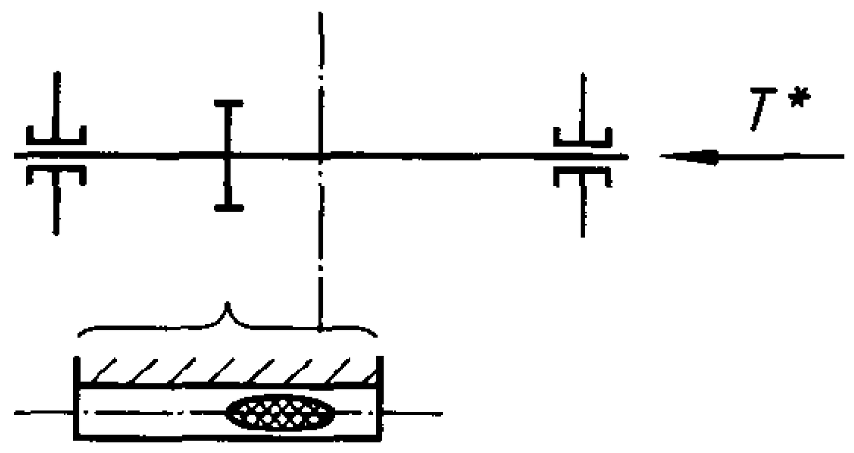
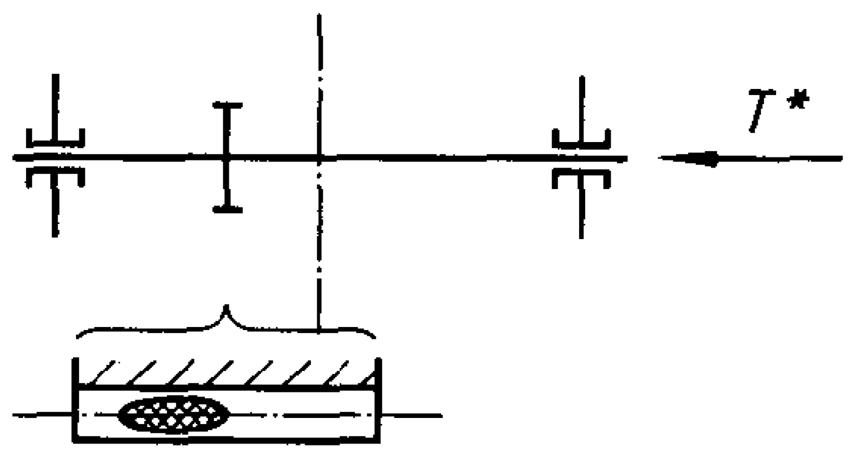
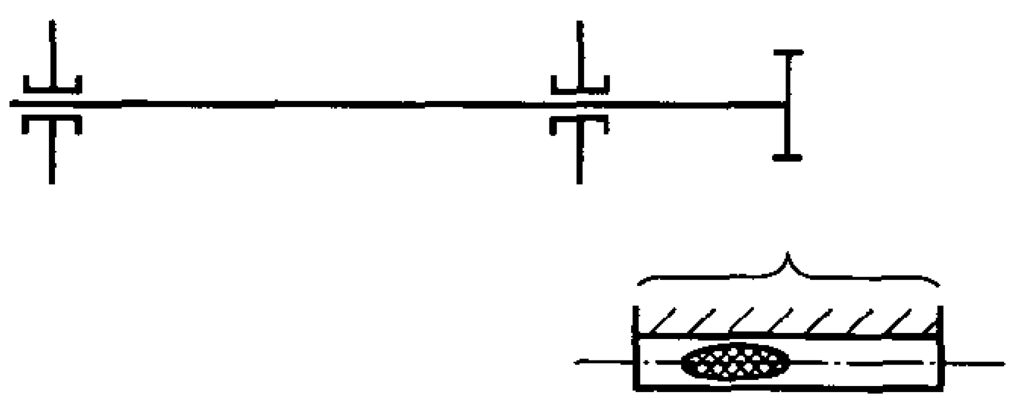
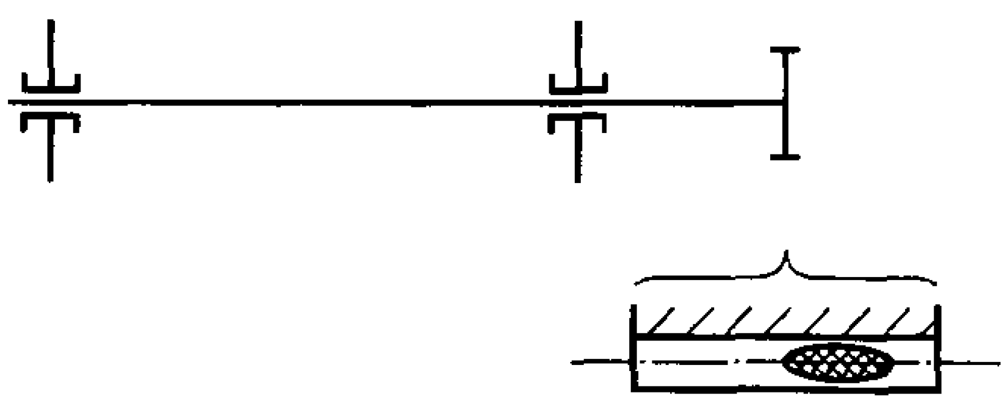
In Figure 2, the pinions shown in dashed lines indicate those helices of double helical gears which have the lower value of f_{sh} and normal shrink fit (for a normal shrink fit, the supporting effect is negligible). The root diameter shall be somewhat greater than the shaft diameter.

5.7.7 Misalignment due to manufacturing inaccuracies, f_{ma}

The misalignment due to manufacturing inaccuracies f_{ma} equals the helix tolerance $f_{H\beta}$:

$$f_{ma} = f_{H\beta} \tag{32}$$

The greater wheel and pinion value should be used. Theoretically, it is possible that manufacturing tolerances of pinion, wheel and shaft alignment may sum up to the worst case. Load distribution should be verified by, for example, contact pattern control.

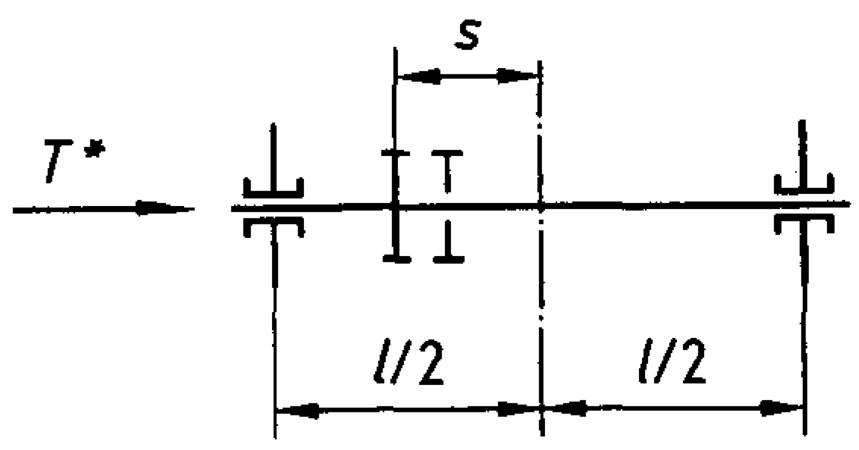
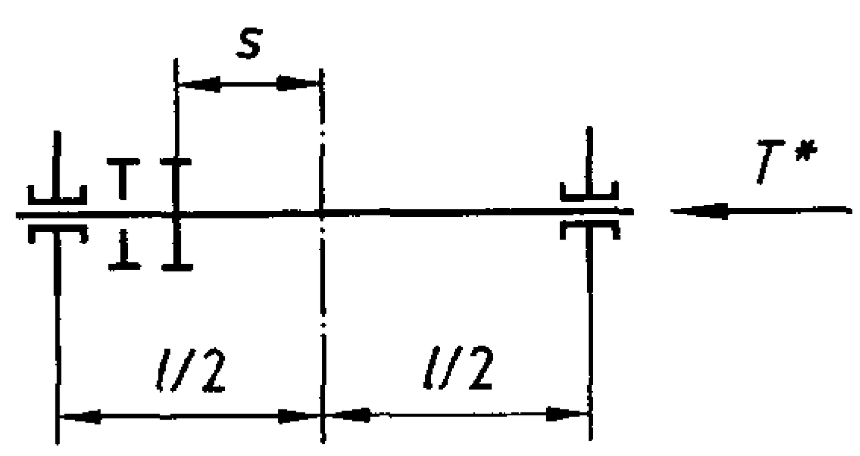
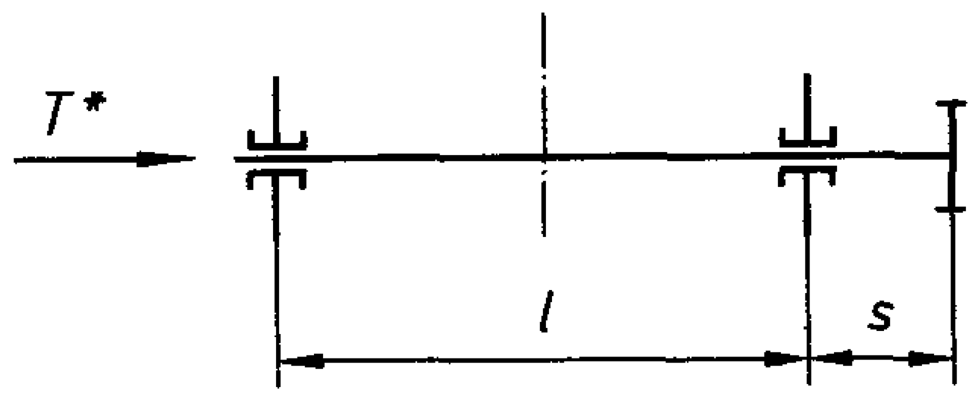
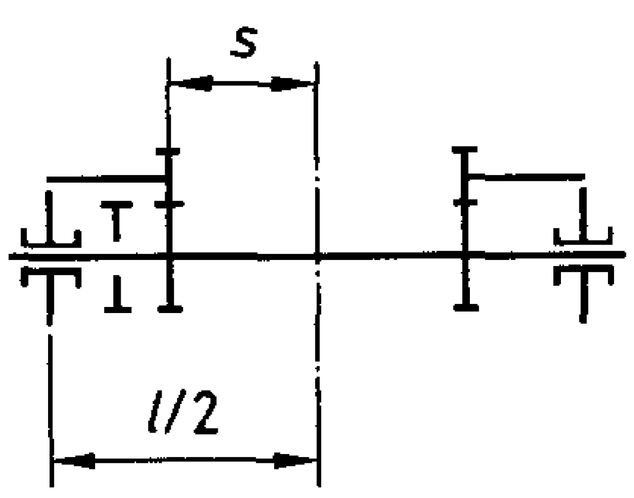
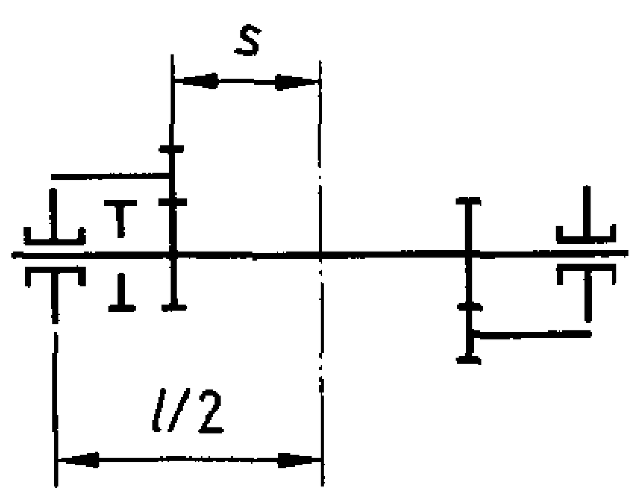
Figure	Position of contact pattern	Determination of $F_{\beta x}$
a)	Contact pattern lies towards mid-bearing span 	$F_{\beta x}$ in accordance with equation (28) (compensatory)
b)	Contact pattern lies away from mid-bearing span 	$F_{\beta x}$ in accordance with equation (27) (additive)
c)	Contact pattern lies towards mid-bearing span 	$F_{\beta x}$ in accordance with equation (27) $\lvert K' \rvert \cdot l \cdot s/d_1^2 \, (d_1/d_{sh})^4 \leqslant B^*$ (additive) $F_{\beta x}$ in accordance with equation (28) $\lvert K' \rvert \cdot l \cdot s/d_1^2 \, (d_1/d_{sh})^4 > B^*$ (compensatory)
d)	Contact pattern lies away from mid-bearing span 	$F_{\beta x}$ in accordance with equation (27) $\lvert K' \rvert \cdot l \cdot s/d_1^2 \, (d_1/d_{sh})^4 \geqslant B^* - 0{,}3$ (additive) $F_{\beta x}$ in accordance with equation (28) $\lvert K' \rvert \cdot l \cdot s/d_1^2 \, (d_1/d_{sh})^4 < B^* - 0{,}3$ (compensatory)
e)	Contact pattern lies towards the bearing 	$F_{\beta x}$ in accordance with equation (27) (additive)
f)	Contact pattern lies away from the bearing 	$F_{\beta x}$ in accordance with equation (28) (compensatory)

a) to d) are the most common mounting arrangement with pinion between bearings; e) and f) have overhung pinions;

T^* is the input or output torqued end, not dependent on direction of rotation;

B^* = 1 for spur and single helical gears; 1,5 for double helical gears. The peak load intensity occurs on the helix near to the torqued end. See also 7.6.2.

Figure 1 — Rules for determination of $F_{\beta x}$ with regard to contact pattern position

Factor K' with stiffening[a]	Factor K' without stiffening[a]	Figure	Arrangement
0,48	0,8	a)	 with $s/l < 0{,}3$
−0,48	−0,8	b)	 with $s/l < 0{,}3$
1,33	1,33	c)	 with $s/l < 0{,}5$
−0,36	−0,6	d)	 with $s/l < 0{,}3$
−0,6	−1,0	e)	 with $s/l < 0{,}3$

T^* is the input or output torqued end, not dependent on direction of rotation.

The dashed line indicates the less deformed helix of a double helical gear.

Determine f_{sh} from the diameter in the gaps of double helical gearing mounted centrally between bearings.

[a] When $d_1/d_{sh} \geqslant 1{,}15$, stiffening is assumed; when $d_1/d_{sh} < 1{,}15$, there is no stiffening; furthermore, scarcely any or no stiffening at all is to be expected when a pinion slides on a shaft and feather key or a similar fitting, nor when normally shrink fitted.

Figure 2 — Constant K' to be substituted in equations (30) and (31) for the calculation of f_{sh}

5.7.8 Running-in allowance, y_β

a) For St, St (cast), V, GGG (perl., bai.) and GTS (perl.)[7]:

$$y_\beta = \frac{320}{\sigma_{\mathrm{H\,lim}}} F_{\beta x} \tag{33}$$

with $y_\beta \leqslant F_{\beta x}$

when $v \leqslant 5$ m/s:	no restriction
when 5 m/s $< v \leqslant 10$ m/s:	the upper limit is $y_\beta = 25\,600/\sigma_{\mathrm{H\,lim}}$, corresponding to $F_{\beta x} = 80$ µm
when $v > 10$ m/s:	the upper limit is $y_\beta = 12\,800/\sigma_{\mathrm{H\,lim}}$, corresponding to $F_{\beta x} = 40$ µm

b) For GG and GGG (ferr.)[7]:

$$y_\beta = 0{,}55\, F_{\beta x} \tag{34}$$

when $v \leqslant 5$ m/s:	no restriction
when 5 m/s $< v \leqslant 10$ m/s:	the upper limit is $y_\beta = 45$ µm, corresponding to $F_{\beta x} = 80$ µm
when $v > 10$ m/s:	the upper limit is $y_\beta = 22$ µm, corresponding to $F_{\beta x} = 40$ µm

c) For Eh, IF, NT (nitr.), NV (nitr.) and NV (nitrocar.)[7]:

$$y_\beta = 0{,}15\, F_{\beta x} \tag{35}$$

For all speeds, the upper limit is $y_\beta = 6$ µm, corresponding to $F_{\beta x} = 40$ µm

When the material of the pinion differs from that of the wheel, $y_{\beta 1}$ for the pinion and $y_{\beta 2}$ for the wheel are to be determined separately.

The mean of the values

$$y_\beta = \frac{\left(y_{\beta 1} + y_{\beta 2}\right)}{2} \tag{36}$$

is used for the calculation.

5.8 Face load factor, $K_{\mathrm{F}\beta}$

$$K_{\mathrm{F}\beta} = K_{\mathrm{H}\beta}^{N_\mathrm{F}} \tag{37}$$

If $b/h \geqslant 3$ then

$$N_\mathrm{F} = \frac{(b/h)^2}{1 + b/h + (b/h)^2} = \frac{1}{1 + h/b + (h/b)^2} \tag{38}$$

7) See Table 2 for an explanation of the abbreviations used.

If $b/h < 3$ then

$$N_F = 0{,}692\ 3 \tag{39}$$

where

b is the facewidth (see 4.4);

h is the tooth height from tip to root: $h = (d_a - d_f) / 2$.

5.9 Transverse load factors, $K_{H\alpha}$, $K_{F\alpha}$

5.9.1 General

The transverse load factors account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth as follows [8]:

a) Values $K_{H\alpha}$ and $K_{F\alpha}$ for gears with total contact ratio $\varepsilon_\gamma \leqslant 2$

$$K_{H\alpha} = K_{F\alpha} = \frac{\varepsilon_\gamma}{2}\left(0{,}9 + 0{,}4\,\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b}\right) \tag{40}$$

b) Values $K_{H\alpha}$ and $K_{F\alpha}$ for gears with total contact ratio $\varepsilon_\gamma > 2$

$$K_{H\alpha} = K_{F\alpha} = 0{,}9 + 0{,}4\sqrt{\frac{2\,(\varepsilon_\gamma - 1)}{\varepsilon_\gamma}}\,\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b} \tag{41}$$

where the following are to be determined:

c_γ mesh stiffness in accordance with Annex B;

f_{pb} the larger of the base pitch deviations of pinion or wheel should be used; 50 % of this tolerance may be used, when profile modifications compensate for the deflections of the teeth at the actual load level[9];

y_α running-in allowance as specified in 5.9.4;

F_{tH} determinant tangential load in a transverse plane, $F_{tH} = F_t K_A K_v K_{H\beta}$.

5.9.2 Limiting conditions for $K_{H\alpha}$

When, in accordance with equation (40) or (41)

$$K_{H\alpha} > \frac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2} \tag{42}$$

8) Equations (40) and (41) are based on the assumption that the base pitch deviations appropriate to the gear accuracy specified are distributed around the circumference of the pinion and wheel as is consistent with normal manufacturing practice. They do not apply when the gear teeth have some intentional deviation.

9) The base pitch deviation f_{pb} accounts for the total effect of all gear tooth deviations which affect the transverse load factor. If, nevertheless, the profile form deviation $f_{f\alpha}$ is greater than the base pitch deviation, the profile form deviation shall be taken instead of the base pitch deviation.

then for $K_{\mathrm{H}\alpha}$ substitute $\dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2}$ and when $K_{\mathrm{H}\alpha} < 1{,}0$, then for $K_{\mathrm{H}\alpha}$ substitute 1,0 as the limit value.

5.9.3 Limiting conditions for $K_{\mathrm{F}\alpha}$

If, in accordance with equation (40) or (41),

$$K_{\mathrm{F}\alpha} > \frac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon^2} \tag{43}$$

then for $K_{\mathrm{F}\alpha}$ substitute $\dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon}$ and when $K_{\mathrm{F}\alpha} < 1{,}0$, then for $K_{\mathrm{F}\alpha}$ substitute 1,0 as the limit value

where

$$Y_\varepsilon = 0{,}25 + \frac{0{,}75}{\varepsilon_{\alpha\mathrm{n}}} \tag{44}$$

with $\varepsilon_{\alpha\mathrm{n}}$ derived from equation (95).

With limiting values in accordance with equations (42) and (43), the least favourable distribution of load is assumed, implying that the entire tangential load is transferred by only one pair of mating teeth. Furthermore, it is recommended that the accuracy of helical gears be so chosen that $K_{\mathrm{H}\alpha}$ and $K_{\mathrm{F}\alpha}$ are not greater than ε_α. As a consequence, it may be necessary to limit the base pitch deviation tolerances of gears of coarse accuracy grade.

5.9.4 Running-in allowance, y_α

The value y_α is the amount by which the initial base pitch deviation is reduced by running-in from the start of operation; y_α does not account for an allowance due to any extent of running-in as a controlled measure, being part of the production process (e.g. lapping). This adjustment shall be taken into consideration when considering the gear quality.

The running-in allowance y_α may be calculated using equations (45) to (48).

a) For St, St (cast), V, GGG (perl., bai.) and GTS (perl.)[10]:

$$y_\alpha = \frac{160}{\sigma_{\mathrm{H\,lim}}} f_{\mathrm{pb}} \tag{45}$$

when $v \leqslant 5$ m/s:	no restriction
when 5 m/s $< v \leqslant 10$ m/s:	the upper limit of y_α is 12 800/$\sigma_{\mathrm{H\,lim}}$ corresponding to $f_{\mathrm{pb}} = 80$ µm
when $v > 10$ m/s:	the upper limit of y_α is 6 400/$\sigma_{\mathrm{H\,lim}}$ corresponding to $f_{\mathrm{pb}} = 40$ µm

b) For GG and GGG (ferr.)[10]:

$$y_\alpha = 0{,}275 f_{\mathrm{pb}} \tag{46}$$

when $v \leqslant 5$ m/s: no restriction

10) See Table 2 for an explanation of the abbreviations used.

when 5 m/s < $v \leqslant$ 10 m/s: the upper limit of y_α is 22 µm corresponding to f_{pb} = 80 µm

when v > 10 m/s: the upper limit of y_α is 11 µm corresponding to f_{pb} = 40 µm

c) For Eh, IF, NT (nitr.), NV (nitr.) and NV (nitrocar.) [10]:

$$y_\alpha = 0{,}075 f_{pb} \tag{47}$$

for all velocities, but with the restriction: the upper limit of y_α is 3 µm corresponding to f_{pb} = 40 µm

When the materials differ, $y_{\alpha 1}$ should be determined for the pinion material and $y_{\alpha 2}$ for the wheel. The average value is used for the calculation:

$$y_\alpha = \frac{y_{\alpha 1} + y_{\alpha 2}}{2} \tag{48}$$

6 Calculation of surface durability (pitting)

6.1 Basic formulae

6.1.1 General

The calculation of surface durability is based on the contact stress, σ_H, at the pitch point or at the inner (lowest) point of single pair tooth contact. The higher of the two values obtained is used to determine capacity. The values of σ_H and the permissible contact stress, σ_{HP}, shall be calculated separately for wheel and pinion; σ_H shall be less than or equal to σ_{HP}.

6.1.2 Determination of contact stress, σ_H, for the pinion

Contact stress σ_H for the pinion is calculated as

$$\sigma_H = Z_B \sigma_{H0} \sqrt{K_A K_v K_{H\beta} K_{H\alpha}} \leqslant \sigma_{HP} \tag{49}$$

with

$$\sigma_{H0} = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b_H} \frac{u \pm 1}{u}} \tag{50}$$

(use the negative sign for internal gears)

where

σ_{H0} is the nominal contact stress at the pitch point: this is the stress induced in flawless (error-free) gearing by application of static nominal torque;

b_H is the facewidth (see 4.4);

Z_B is the single pair tooth contact factor for the pinion (see 6.2).

6.1.3 Determination of contact stress, σ_H, for the wheel

Contact stress σ_H for the wheel is calculated as

$$\sigma_H = Z_D \sigma_{H0} \sqrt{K_A K_v K_{H\beta} K_{H\alpha}} \leqslant \sigma_{HP} \tag{51}$$

where

Z_D is the single pair tooth contact factor for the wheel (see 6.2).

The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This shall be taken into consideration by substituting $K_\gamma K_A$ for K_A in equation (49) and equation (51) to adjust the average tangential load per mesh as necessary (see clause 5).

6.1.4 Determination of permissible contact stress, σ_{HP}

6.1.4.1 Method

In this International Standard, Method B of ISO 6336-2:1996 is used.

$$\sigma_{HP} = \frac{\sigma_{H\lim} Z_N}{S_{H\min}} Z_L Z_v Z_R Z_W Z_X = \frac{\sigma_{HG}}{S_{H\min}} \tag{52}$$

6.1.4.2 Permissible contact stress (reference), $\sigma_{HP\,ref}$

The permissible contact stress (reference), $\sigma_{HP\,ref}$, shall be derived from equation (52), with $Z_N = 1$ and appropriate values of $\sigma_{H\,lim}$, Z_L, Z_v, Z_R, Z_W, Z_X, $S_{H\,min}$.

6.1.4.3 Permissible contact stress (static), $\sigma_{HP\,stat}$

The permissible contact stress (static), $\sigma_{HP\,stat}$, shall be determined in accordance with equation (52), with $Z_N = Z_{NT}$ for static stress according to 6.8 and appropriate values of $\sigma_{H\,lim}$, Z_L, Z_v, Z_R, Z_W, Z_X, $S_{H\,min}$.

6.1.4.4 Permissible contact stress (10^{10} cycles), $\sigma_{HP\,10}$

The permissible contact stress (10^{10} cycles), $\sigma_{HP\,10}$, shall be determined in accordance with equation (52), with $Z_N = Z_{NT}$ for 10^{10} load cycles according to 6.8 and appropriate values of $\sigma_{H\,lim}$, Z_L, Z_v, Z_R, Z_W, Z_X, $S_{H\,min}$.

6.1.4.5 Permissible contact stress, σ_{HP}, for limited or long life

The limited life range is that where the number of load cycles N_L lies between the value corresponding to the static allowable and the values corresponding to the reference allowable listed in Table 6 (see Figure 3).

The long-life range is that where the number of load cycles N_L lies between the value corresponding to the reference allowable listed in Table 6 and 10^{10} load cycles (see Figure 3).

— σ_{HP} for a given number of load cycles N_L in the limited life range is determined by graphical or calculated interpolation (on a log-log scale) between the value obtained for reference strength in accordance with 6.1.4.2 and the value obtained for static strength in accordance with 6.1.4.3.

— σ_{HP} for a given number of load cycles N_L in the long life range is determined by graphical or calculated interpolation (on a log-log scale) between the value obtained for reference strength in accordance with 6.1.4.2 and the value obtained for 10^{10} load cycles in accordance with 6.1.4.4.

Values of the permissible contact stress, σ_{HP}, for more than 10^{10} load cycles have not been established.

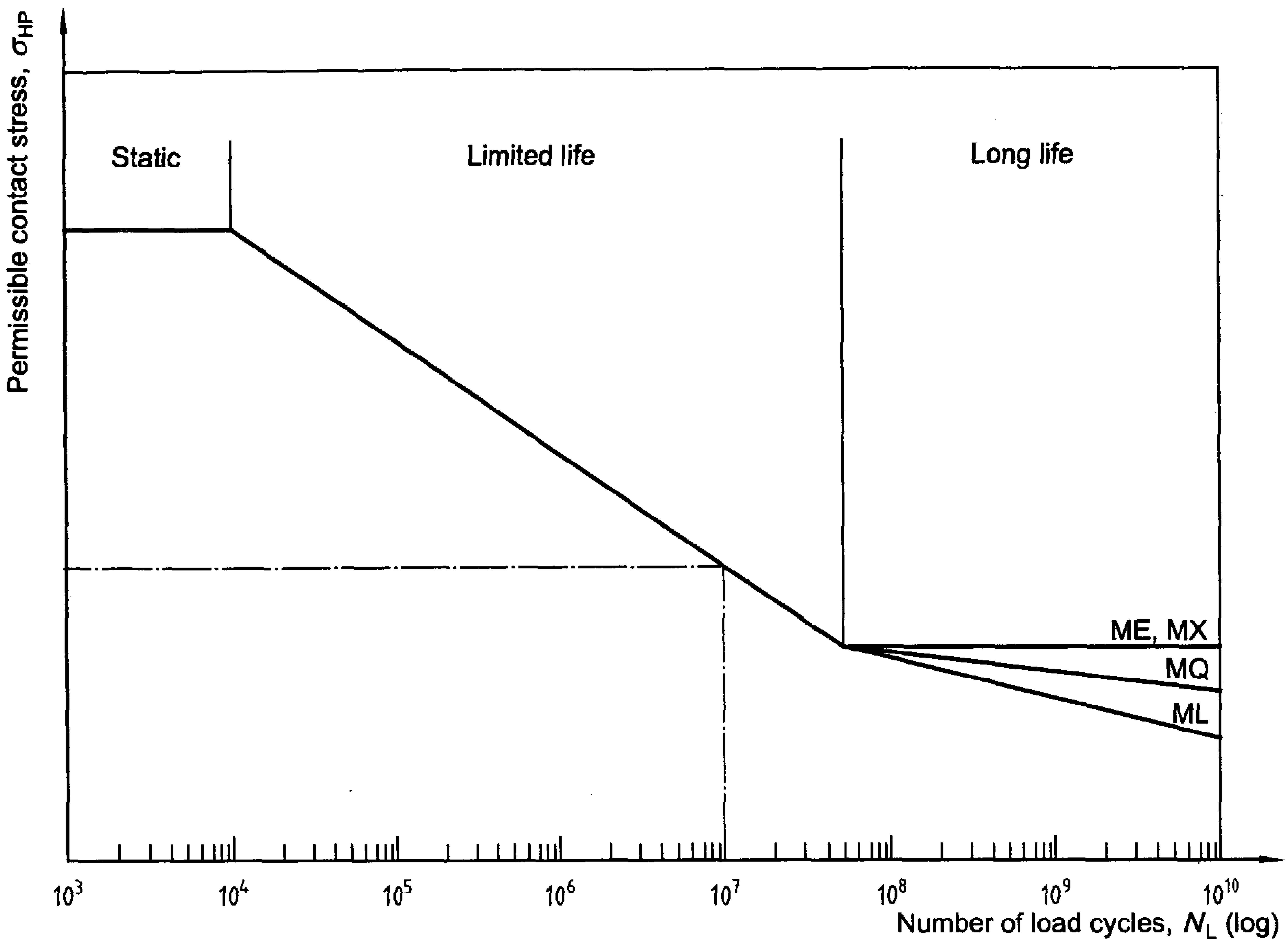


Figure 3 — Graphical determination of permissible contact stress for limited and long life — Example: Permissable contact stress, σ_{HP}, for 10^7 load cycles.

6.1.5 Safety factor for surface durability, S_H

S_H shall be calculated separately for pinion and wheel.

$$S_H = \frac{\sigma_{HG}}{\sigma_H} > S_{H\,min} \tag{53}$$

with σ_{HG} for reference and static stresses, according to equation (52) and 6.1.4; σ_H shall be in accordance with equation (49) for the pinion and with equation (51) for the wheel (see 6.1).

NOTE This is the calculated safety factor with regard to contact stress (Hertzian pressure). The corresponding factor relative to torque capacity is equal to the square of S_H.

For the minimum safety factor for surface durability, $S_{H\,min}$, see 6.12.

6.2 Single pair tooth contact factors, Z_B, Z_D

When $Z_B > 1$ or $Z_D > 1$, the factors Z_B and Z_D are used to transform the contact stress at the pitch point of spur gears to the contact stress at the inner (lowest) limit of single pair tooth contact of the pinion and the wheel, respectively. See the introduction to 6.1.

a) Internal gears

Z_D is always to be taken as unity.

b) Spur gears

Determine M_1 [quotient of $\rho_{rel\ C}$ at the pitch point by $\rho_{rel\ B}$ at the inner limit (lowest point) of single tooth pair contact of the pinion] and M_2 (quotient of $\rho_{rel\ C}$ by $\rho_{rel\ D}$ of the wheel) from

$$M_1 = \frac{\tan \alpha_{wt}}{\sqrt{\left[\sqrt{\frac{d_{a1}^2}{d_{b1}^2} - 1} - \frac{2\pi}{z_1}\right]\left[\sqrt{\frac{d_{a2}^2}{d_{b2}^2} - 1} - (\varepsilon_\alpha - 1)\frac{2\pi}{z_2}\right]}} \quad (54)$$

$$M_2 = \frac{\tan \alpha_{wt}}{\sqrt{\left[\sqrt{\frac{d_{a2}^2}{d_{b2}^2} - 1} - \frac{2\pi}{z_2}\right]\left[\sqrt{\frac{d_{a1}^2}{d_{b1}^2} - 1} - (\varepsilon_\alpha - 1)\frac{2\pi}{z_1}\right]}} \quad (55)$$

(See 6.5.2 for the calculation of the profile contact ratio ε_α)

If $M_1 > 1$ then $Z_B = M_1$ if $M_1 \leqslant 1$ then $Z_B = 1{,}0$

if $M_2 > 1$ then $Z_D = M_2$ if $M_2 \leqslant 1$ then $Z_D = 1{,}0$

c) Helical gears with $\varepsilon_\beta \geqslant 1$

$Z_B = Z_D = 1$

d) Helical gears with $\varepsilon_\beta < 1$

Z_B and Z_D are determined by linear interpolation between the values for spur and helical gearing with $\varepsilon_\beta \geqslant 1$:

$$Z_B = M_1 - \varepsilon_\beta (M_1 - 1); \; Z_B \geqslant 1$$
$$Z_D = M_2 - \varepsilon_\beta (M_2 - 1); \; Z_D \geqslant 1 \quad (56)$$

If Z_B or Z_D are set to unity, the contact stresses calculated using equations (49) or (51) are the values for the contact stress at the pitch cylinder.

The methods in 6.2 apply to the calculation of contact stress when the pitch point lies in the path of contact. If the pitch point C is determinant and lies outside the path of contact, then Z_B and/or Z_D are to be determined for contact at the adjacent tip circle. For helical gears when ε_β is less than 1,0, Z_B and Z_D are to be determined by linear interpolation between the values (determined at the pitch point or at the adjacent tip circle as appropriate) for spur gears and those helical gears with $\varepsilon_\beta \geqslant 1$.

6.3 Zone factor, Z_H

The zone factor, Z_H, accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point and transforms the tangential force at the reference cylinder to normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2 \cos \beta_b \cos \alpha_{wt}}{\cos^2 \alpha_t \sin \alpha_{wt}}} \quad (57)$$

6.4 Elasticity factor, Z_E

The elasticity factor, Z_E, takes into account the influences of the material properties E (modulus of elasticity) and ν (Poisson's ratio) on the contact stress.

Numerical values are given in Table 5.

Table 5 — Elasticity factor, Z_E, for some material combinations, mean values

Wheel 1			Wheel 2			
Material[a]	**Modulus of elasticity** N/mm^2	**Poisson's ratio** ν	**Material**[a]	**Modulus of elasticity** N/mm^2	**Poisson's ratio** ν	Z_E $\sqrt{\text{N/mm}^2}$
St, V, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.)	206 000	0,3	St, V, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.)	206 000	0,3	189,8
			St (cast)	202 000		188,9
			GGG (perl., bai., ferr.)	173 000		181,4
			GTS (perl.)	170 000		180,5
			GG	126 000 to 118 000		165,4 to 162,0
St (cast)	202 000		St (cast)	202 000		188,0
			GGG (perl., bai., ferr.)	173 000		180,5
			GTS (perl.)	170 000		179,7
			GG)	118 000		161,4
GGG (perl., bai., ferr.)	173 000		GGG (perl., bai., ferr.)	173 000		173,9
			GTS (perl.)	170 000		173,2
			GG	118 000		156,6
GTS (perl.)	170 000		GTS (perl.)	170 000		172,4
			GG	118 000		156,1
GG	126 000 to 118 000		GG	118 000		146,0 to 143,7

[a] See Table 2 for an explanation of the abbreviations used.

6.5 Contact ratio factor, Z_ε

6.5.1 General

The contact ratio factor, Z_ε, accounts for the influence of the transverse contact and overlap ratios on the surface load capacity of cylindrical gears.

a) Spur gears:

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}} \tag{58}$$

The conservative value of $Z_\varepsilon = 1{,}0$ may be chosen for spur gears having a contact ratio of less than 2,0.

b) Helical gears:

If $\varepsilon_\beta < 1$ then

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}\left(1-\varepsilon_\beta\right)+\frac{\varepsilon_\beta}{\varepsilon_\alpha}} \tag{59}$$

If $\varepsilon_\beta \geqslant 1$ then

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \tag{60}$$

6.5.2 Transverse contact ratio, ε_α

$$\varepsilon_\alpha = g_\alpha / p_{bt} \tag{61}$$

where

length of path of contact:

$$g_\alpha = \frac{1}{2}\left[\sqrt{d_{a1}^2 - d_{b1}^2} \pm \sqrt{d_{a2}^2 - d_{b2}^2}\right] - a \sin \alpha_{wt} \tag{62}$$

(use the positive sign for external gears, the negative sign for internal gears)

transverse base pitch:

$$p_{bt} = m_t \, \pi \cos \alpha_t \tag{63}$$

Equation (62) is only valid if the path of contact is effectively limited by the tip circle of the pinion and the wheel and not, for example, by undercut tooth profiles.

6.5.3 Overlap ratio, ε_β

$$\varepsilon_\beta = \frac{b_H \sin \beta}{\pi m_n} \tag{64}$$

6.6 Helix angle factor, Z_β

The helix angle factor, Z_β, takes account of the influence on surface stress of the helix angle.

$$Z_\beta = \sqrt{\cos \beta} \tag{65}$$

6.7 Allowable stress numbers (contact), $\sigma_{H\,lim}$

ISO 6336-5 provides information on commonly used gear materials, methods of heat treatment and the influence of gear quality on values for allowable stress numbers, $\sigma_{H\,lim}$, derived from test results of standard reference test gears.

Also see ISO 6336-5 for requirements concerning material and heat treatment for qualities ML, MQ, ME and MX. Material quality MQ shall be chosen for industrial gears, unless otherwise agreed.

6.8 Life factor, Z_{NT}

Method B of ISO 6336-3:1996 is used in this International Standard. Values for Z_{NT} are as listed in Table 6.

Table 6 — Life factor, Z_{NT}

Material[a]	Number of load cycles	Life factor Z_{NT}
St, St (cast), V, GGG (perl. bain.), GTS (perl.), Eh, IF — Only when a certain degree of pitting is permissible	$N_L \leqslant 6 \times 10^5$ (static)	1,6
	$N_L = 10^7$	1,3
	$N_L = 10^9$ (reference)	1,0
	$N_L = 10^{10}$	ME, MX: 1,0[b]
		MQ: 0,92
		ML: 0,85
St, St (cast), V, GGG (perl. bain.), GTS (perl.), Eh, IF — No pitting is permissible	$N_L \leqslant 10^5$ (static)	1,6
	$N_L = 5 \times 10^7$ (reference)	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	$N_L \leqslant 10^5$ (static)	1,3
	$N_L = 2 \times 10^6$ (reference)	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85
NV (nitrocar.)	$N_L \leqslant 10^5$ (static)	1,1
	$N_L = 2 \times 10^6$ (reference)	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85

a See Table 2 for an explanation of abbreviations used.

b Optimum lubrication, manufacturing and experience supposed.

6.9 Influences on lubrication film formation, Z_L, Z_v and Z_R

6.9.1 General

As described in ISO 6336-2, Z_L accounts for the influence of nominal viscosity of the lubricant, Z_v, for the influence of tooth-flank velocities and Z_R for the influence of surface roughness on the formation of the lubricant film in the contact zone. Method C of ISO 6336-2:1996 is used in this International Standard.

The viscosity of the lubricant shall be chosen to suit the operating conditions (pitch line velocity, loading, size), so that the product $Z_L Z_v$ will be approximately 1,0.

Depending on the flank surface roughness associated with the manufacturing process used, it is assumed that the remaining factor Z_R is of almost constant value.

6.9.2 Product $Z_L Z_v Z_R$ for reference strength and long life

— For gears which are hobbed, shaped or planed, or which do not meet the following three conditions:

$$Z_L Z_v Z_R = 0{,}85 \tag{66}$$

— For gears with lapped, ground or shaved teeth and mean relative peak-to-valley roughness, Rz_{10}:

$$Rz_{10} = \frac{Rz_1 + Rz_2}{2} \sqrt[3]{\frac{20(d_{b1} + d_{b2})}{\tan\alpha_{wt} d_{b1} d_{b2}}} > 4\ \mu m \tag{67}$$

$$Z_L Z_v Z_R = 0{,}92 \tag{68}$$

— For gear pairs in which one gear is hobbed, shaped or planed and the mating gear is ground or shaved, with $Rz_{10} \leqslant 4\ \mu m$:

$$Z_L Z_v Z_R = 0{,}92 \tag{69}$$

— For ground or shaved gearing with $Rz_{10} \leqslant 4\ \mu m$:

$$Z_L Z_v Z_R = 1{,}0 \tag{70}$$

6.9.3 Product $Z_L Z_v Z_R$ for static strength

$Z_L Z_v Z_R = 1{,}0$ applies for static strength in all cases.

6.10 Work hardening factor, Z_W

As described in ISO 6336-2, the work hardening factor Z_W takes account of the increased surface durability due to meshing a steel wheel (structural steel, through-hardened steel) with a pinion which is significantly ($\approx$ 200 HB or more) harder than the wheel and with smooth tooth flanks ($Rz \leqslant 6\ \mu m$, otherwise effects of wear are not covered by this International Standard). Method B of ISO 6336-2:1996 is applied, as follows:

if HB < 130 then

$$Z_W = 1{,}2 \tag{71}$$

if 130 $\leqslant$ HB $\leqslant$ 470 then

$$Z_W = 1{,}2 - \frac{HB - 130}{1\,700} \tag{72}$$

If HB > 470 then

$$Z_W = 1{,}0 \tag{73}$$

where HB is the Brinell hardness of the tooth flanks of the softer gear of the pair.

6.11 Size factor, Z_X

By means of Z_X, account is taken of statistical evidence indicating that the stress levels at which fatigue damage occurs decrease with an increase of component size (larger number of weak points in structure), as a consequence of the influence on subsurface defects of the smaller stress gradients which occur (theoretical stress analysis) and the influence of size on material quality (effect on forging process, variations in structure, etc.). Important influence parameters are:

a) material quality (furnace charge, cleanliness, forging);

b) heat treatment, depth of hardening, distribution of hardening;

c) radius of flank curvature;

d) module — in the case of surface hardening, depth of hardened layer relative to the size of teeth (core supporting-effect).

For through-hardened gears and for surface-hardened gears with adequate case depth relative to tooth size and radius of relative curvature, the size factor, Z_X, is taken to be 1,0.

6.12 Minimum safety factor (pitting), $S_{H\,min}$

For general aspects concerning safety factors, see clause 4; for calculation of actual safety factor (pitting), S_H, see 6.1.5. If not otherwise agreed between manufacturer and user, the following minimum safety factor (pitting), $S_{H\,min}$, shall be applied:

$$S_{H\,min} = 1,0 \qquad (74)$$

7 Calculation of tooth bending strength

7.1 Basic formulae

7.1.1 General

As described in ISO 6336-3, the maximum tensile stress at the tooth root may not exceed the permissible bending stress for the material. This is the basis for rating the bending strength of gear teeth.

The actual tooth-root stress, σ_F, and the permissible bending stress, σ_{FP}, shall be calculated separately for pinion and wheel; σ_F shall be less than σ_{FP}.

7.1.2 Determination of tooth root stress, σ_F

In this International Standard, Method B of ISO 6336-3:1996 is used.

Tooth root stress σ_F is calculated

$$\sigma_F = \sigma_{F0} K_A K_v K_{F\beta} K_{F\alpha} \leqslant \sigma_{FP} \qquad (75)$$

with

$$\sigma_{F0} = \frac{F_t}{b_F m_n} Y_F Y_S Y_\beta \qquad (76)$$

where

σ_{F0} is the nominal tooth-root stress: this is the maximum local tensile stress produced at the tooth-root when an error-free gear pair is loaded by the static nominal torque;

b_F is the facewidth (see 4.4).

The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by substituting $K_\gamma K_A$ for K_A in equation (75) to adjust the average tangential load per mesh as necessary; see clause 5.

7.1.3 Determination of permissible tooth root stress, σ_{FP}

7.1.3.1 General

Equation (77) shall be used for the determination of the permissible tooth root stress.

$$\sigma_{FP} = \frac{\sigma_{FE} Y_N}{S_{Fmin}} Y_{\delta\,rel\,T} Y_{R\,rel\,T} Y_X = \frac{\sigma_{FG}}{S_{Fmin}} \tag{77}$$

7.1.3.2 Permissible tooth root stress (reference), $\sigma_{FP\,ref}$

To evaluate the permissible tooth root stress (reference), $\sigma_{FP\,ref}$, use equation (77) with $Y_N = 1$ and appropriate values of σ_{FE}, $Y_{\delta\,rel\,T}$, $Y_{R\,rel\,T}$, Y_X and $S_{F\,min}$.

7.1.3.3 Permissible tooth root stress (static), $\sigma_{FP\,stat}$

To evaluate the permissible tooth root stress (static), $\sigma_{FP\,stat}$, use equation (77) with $Y_N = Y_{NT}$ for static stress according to 7.5 and appropriate values of σ_{FE}, $Y_{\delta\,rel\,T}$, $Y_{R\,rel\,T}$, Y_X and $S_{F\,min}$.

7.1.3.4 Permissible tooth root stress (10^{10} load cycles), σ_{FP10}

To evaluate the permissible tooth root stress (10^{10} load cycles), σ_{FP10}, use equation (77) with $Y_N = Y_{NT}$ for 10^{10} cycles according to 7.5 and appropriate values of σ_{FE}, $Y_{\delta\,rel\,T}$, $Y_{R\,rel\,T}$, Y_X and $S_{F\,min}$.

7.1.3.5 Permissible tooth root stress, σ_{FP}, for limited or long life

The limited life range is that where the number of load cycles N_L lies between the value corresponding to the static allowable and 3×10^6 load cycles.

The long life range is that where the number of load cycles N_L lies between 3×10^6 and 10^{10} load cycles.

— σ_{FP} for a given number of load cycles N_L in the limited life range is determined by graphical or calculated interpolation (on a log-log scale) between the value obtained for reference strength in accordance with 7.1.3.2 and that obtained for static strength in accordance with 7.1.3.3.

— σ_{FP} for a given number of load cycles N_L in the long life range is determined by graphical or calculated interpolation (on a log-log scale) between the value obtained for reference strength in accordance with 7.1.3.2 and that obtained for 10^{10} load cycles in accordance with 7.1.3.4.

Values of the permissible tooth root stress, σ_{FP}, for more than 10^{10} cycles have not been established.

7.1.4 Safety factor for bending strength, S_F

The factor S_F shall be calculated from the following equation:

$$S_F = \frac{\sigma_{FG}}{\sigma_F} \geqslant S_{F\,min} \tag{78}$$

S_F is calculated separately for the pinion and wheel, with σ_{FG} calculated in accordance with equation (77) and 7.1.3, and σ_F from equation (75).

More information on safety factor and probability of failure can be found in ISO 6336-1:1996, 4.1.3. For the minimum safety factor for bending strength, $S_{F\,min}$, see 7.9.

7.2 Form factor, Y_F, and stress correction factor, Y_S

7.2.1 General

These are the factors by means of which the influence of tooth form on nominal bending stress is taken into account. Y_F and Y_S are determined separately for pinion and wheel. For more information see ISO 6336-3.

For helical gears, Y_F is determined for the equivalent virtual spur gear. See 7.2.2.4 for the parameters for virtual spur gears.

The equations given below apply for all basic rack tooth profiles, with or without undercut, but with the following restrictions:

a) the contact point of the 30 ° tangent lies on the tooth-root fillet;

b) the basic rack profile of the gear has a root fillet;

c) the teeth were generated using tools such as hobs or planer-cutters having rack form teeth.

7.2.2 Determination of Y_F

7.2.2.1 General

The form factor, Y_F, is determined from the normal chordal dimension s_{Fn} of the tooth-root critical section and the bending moment arm h_{Fe} relevant to load application at the external gear tooth tip using the following equation.

$$Y_F = \frac{\dfrac{6 h_{Fe}}{m_n} \cos \alpha_{Fen}}{\left(\dfrac{s_{Fn}}{m_n}\right)^2 \cos \alpha_n} \tag{79}$$

7.2.2.2 External gearing

If the tip of the tooth has been rounded or chamfered, it is necessary to replace the tip diameter d_a in the calculation, by d_{Na} the "effective tip diameter"; d_{Na} is the diameter of a circle near the tip cylinder, containing limits of the usable gear flanks.

Firstly, determine the auxiliary values E, G, and H:

$$E = \frac{\pi}{4} m_n - h_{fP} \tan \alpha_n + \frac{s_{pr}}{\cos \alpha_n} - (1 - \sin \alpha_n) \frac{\rho_{fP}}{\cos \alpha_n} \tag{80}$$

where

$s_{pr} = pr - q$ (see Figure 4)

s_{pr} = 0 when gears are not undercut (see Figure 4)

$$G = \frac{\rho_{fP}}{m_n} - \frac{h_{fP}}{m_n} + x \tag{81}$$

$$H = \frac{2}{z_n}\left(\frac{\pi}{2} - \frac{E}{m_n}\right) - \frac{\pi}{3} \tag{82}$$

Next, use G and H together with $\theta = \pi/6$ as a seed value (on the right-hand side) in equation (83).

$$\theta = \frac{2G}{z_n} \tan\theta - H \tag{83}$$

Use the newly calculated θ and apply equation (83) again. Continue using equation (83) until there is no significant change in successive values of θ. Generally, the function converges after two or three iterations. Use this final value of θ in equations (84), (85), and (86).

Tooth-root normal chord, s_{Fn}:

$$\frac{s_{Fn}}{m_n} = z_n \sin\left(\frac{\pi}{3} - \theta\right) + \sqrt{3}\left(\frac{G}{\cos\theta} - \frac{\rho_{fP}}{m_n}\right) \tag{84}$$

Radius of root fillet, ρ_F:

$$\frac{\rho_F}{m_n} = \frac{\rho_{fP}}{m_n} + \frac{2G^2}{\cos\theta\,(z_n \cos^2\theta - 2G)} \tag{85}$$

Bending moment arm, h_{Fe}:

$$\frac{h_{Fe}}{m_n} = 0{,}5\left[(\cos\gamma_e - \sin\gamma_e \tan\alpha_{Fen})\frac{d_{en}}{m_n} - z_n \cos\left(\frac{\pi}{3} - \theta\right) - \frac{G}{\cos\theta} + \frac{\rho_{fP}}{m_n}\right] \tag{86}$$

See 7.2.2.4 for parameters for the virtual gear.

7.2.2.3 Internal gearing

It is assumed that the value of the form factor of a special rack can be substituted as an approximate value of the form factor of an internal gear. The profile of such a rack should be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart gear of the internal gear. The tip load angle is α_n.

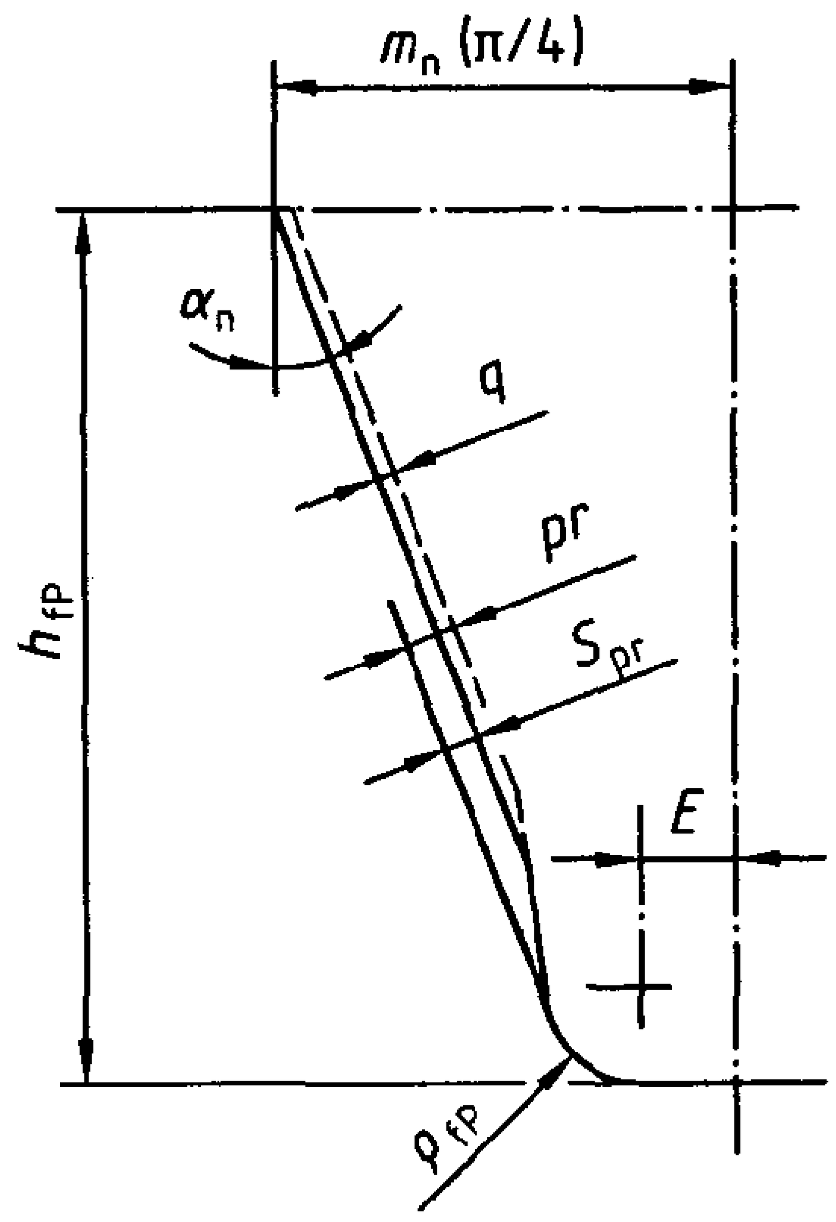


Figure 4 — Basic rack profile of protuberance rack

The values to be used in equation (79) are determined as follows.

Tooth-root normal chord, s_{Fn2}:

$$\frac{s_{Fn2}}{m_n} = 2\left[\frac{\pi}{4} + \frac{h_{fP2} - \rho_{fP2}}{m_n}\tan\alpha_n + \frac{\rho_{fP2} - s_{pr}}{m_n \cos\alpha_n} - \frac{\rho_{fP2}}{m_n}\cos\frac{\pi}{6}\right] \qquad (87)$$

where

ρ_{fP2} is the tool radius (see below)

Bending moment arm, h_{Fe2}:

$$\frac{h_{Fe2}}{m_n} = \frac{d_{en2} - d_{fn2}}{2\,m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fP2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2m_n}\right)\tan\alpha_n\right]\tan\alpha_n - \frac{\rho_{fP2}}{m_n}\left(1 - \sin\frac{\pi}{6}\right) \qquad (88)$$

with

d_{en2} to be derived from equation (100) with parameters having 2 added to the subscripts

d_{fn2} to be derived in the same way as d_{an} [equation (99); note that $d_{fn2} - d_{f2} = d_{n2} - d_2$]

and

$$h_{fP2} = \frac{d_{n2} - d_{fn2}}{2} \qquad (89)$$

Root fillet radius ρ_{F2}:

When the root fillet radius of an internal gear tooth, ρ_{F2}, is known, it shall be used for ρ_{fP2}. When it is not known, the following approximation may be used:

$$\rho_{F2} = \rho_{fP2} = 0{,}15 m_n \qquad (90)$$

This shall be validated.

$$\rho_{\text{fP2}} = \frac{h_{\text{f2}} - h_{\text{Nf2}}}{1 - \sin\alpha} = \frac{d_{\text{Nf2}} - d_{\text{f2}}}{2(1 - \sin\alpha_{\text{n}})} \quad (91)$$

where

d_{Nf2} represents the diameter of a circle near the tooth-roots, containing the limits of the usable flanks of an internal gear or the larger external gear of a mating pair. For internal gearing, diameters have a negative sign.

7.2.2.4 Parameters for virtual gears

$$\beta_{\text{b}} = \arccos\sqrt{1 - (\sin\beta\cos\alpha_{\text{n}})^2} = \arcsin(\sin\beta\cos\alpha_{\text{n}}) \quad (92)$$

$$z_{\text{n}} = \frac{z}{\cos^2\beta_{\text{b}}\cos\beta} \quad (93)$$

Approximation:

$$z_{\text{n}} \approx \frac{z}{\cos^3\beta} \quad (94)$$

$$\varepsilon_{\alpha\text{n}} = \frac{\varepsilon_\alpha}{\cos^2\beta_{\text{b}}} \quad (95)$$

$$d_{\text{n}} = \frac{d}{\cos^2\beta_{\text{b}}} = m_{\text{n}} z_{\text{n}} \quad (96)$$

$$p_{\text{bn}} = \pi\, m_{\text{n}} \cos\alpha_{\text{n}} \quad (97)$$

$$d_{\text{bn}} = d_{\text{n}} \cos\alpha_{\text{n}} \quad (98)$$

$$d_{\text{an}} = d_{\text{n}} + d_{\text{a}} - d \quad (99)$$

$$d_{\text{en}} = 2\,\frac{z}{|z|}\sqrt{\left[\sqrt{\left(\frac{d_{\text{an}}}{2}\right)^2 - \left(\frac{d_{\text{bn}}}{2}\right)^2} - \frac{\pi\, d\cos\beta\cos\alpha_{\text{n}}}{|z|}(\varepsilon_{\alpha\text{n}} - 1)\right]^2 + \left(\frac{d_{\text{bn}}}{2}\right)^2} \quad (100)$$

The number of teeth z is positive for external gears and negative for internal gears (see footnote a in Table 1).

$$\alpha_{\text{en}} = \arccos\left(\frac{d_{\text{bn}}}{d_{\text{en}}}\right) \quad (101)$$

$$\gamma_{\text{e}} = \frac{0{,}5\pi + 2\tan\alpha_{\text{n}} x}{z_{\text{n}}} + \text{inv}\,\alpha_{\text{n}} - \text{inv}\,\alpha_{\text{en}} \quad (102)$$

$$\alpha_{\text{Fen}} = \alpha_{\text{en}} - \gamma_{\text{e}} = \tan\alpha_{\text{en}} - \text{inv}\,\alpha_{\text{n}} - \frac{0{,}5\pi + 2\tan\alpha_{\text{n}} x}{z_{\text{n}}} \quad (103)$$

7.2.3 Determination of Y_{S}

The stress correction factor Y_{S} is calculated using equation (104), which is applicable in the range $1 \leqslant q_{\text{s}} < 8$.

$$Y_S = (1,2 + 0,13\,L)\,q_s^{[1/(1,21 + 2,3/L)]} \quad (104)$$

where

$$L = \frac{s_{Fn}}{h_{Fe}} \quad (105)$$

with

s_{Fn} from equation (84) for external gears and equation (87) for internal gears;

h_{Fe} from equation (86) for external gears and equation (88) for internal gears.

$$q_s = \frac{s_{Fn}}{2\rho_F} \quad (106)$$

with

ρ_F from equation (85) for external gears and equation (91) for internal gears.

7.3 Helix angle factor, Y_β

The tooth-root stress of a virtual spur gear, calculated as a preliminary value, is converted by means of the helix factor Y_β to that of the corresponding helical gear. By this means, the oblique orientation of the lines of mesh contact is taken into account (lesser tooth-root stress).

If $\varepsilon_\beta > 1$ and $\beta \leqslant 30°$ then

$$Y_\beta = 1 - \frac{\beta}{120°} \quad (107)$$

If $\varepsilon_\beta > 1$ and $\beta > 30°$ then

$$Y_\beta = 0,75 \quad (108)$$

If $\varepsilon_\beta \leqslant 1$ and $\beta \leqslant 30°$ then

$$Y_\beta = 1 - \varepsilon_\beta \frac{\beta}{120°} \quad (109)$$

If $\varepsilon_\beta \leqslant 1$ and $\beta > 30°$ then

$$Y_\beta = 1 - 0,25\,\varepsilon_\beta \quad (110)$$

7.4 Tooth-root reference strength, σ_{FE}

ISO 6336-5 provides information on values of σ_{Flim} and σ_{FE}, for the more popular gear materials. The requirements for heat treatment processes and material quality for quality grades ML, MQ and ME are also included.

The quality MQ is used for industrial gears unless otherwise agreed. Method B from ISO 6336-3:1996 is used in this International Standard.

7.5 Life Factor, Y_{NT}

Method B from ISO 6336-3:1996 is used in this International Standard. Values for Y_{NT} are given in Table 7.

Table 7 — Life factor, Y_{NT}

Material[a]	Number of load cycles	Life factor Y_{NT}
V, GGG (perl. bain.), GTS (perl.)	$N_L \leqslant 10^4$ (static)	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	ME, MX: 1,0[b]
		MQ: 0,92
		ML: 0,85
Eh, IF (root)	$N_L \leqslant 10^3$ (static)	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85
St, St (cast), NT (nitr.), NV (nitr.), GG, GGG (ferr.)	$N_L \leqslant 10^3$ (static)	1,6
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85
NV (nitrocar.)	$N_L \leqslant 10^3$ (static)	1,1
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	ME: 1,0[b]
		MQ: 0,92
		ML: 0,85

[a] See Table 2 for an explanation of the abbreviations used.

[b] Optimum manufacturing and experience supposed.

7.6 Relative notch sensitivity factor, $Y_{\delta\,\mathrm{rel\,T}}$

7.6.1 General

$Y_{\delta\,\mathrm{rel\,T}}$ indicates, approximately, the overstress tolerance of the material in the root fillet region. In this International Standard, Method B of ISO 6336-3:1996 is used.

7.6.2 $Y_{\delta\,\mathrm{rel\,T}}$ for reference and long life stresses

$Y_{\delta\,\mathrm{rel\,T}}$ can be calculated using equation (111).

$$Y_{\delta\,\mathrm{rel\,T}} = \frac{Y_\delta}{Y_{\delta\mathrm{T}}} = \frac{1+\sqrt{\rho'\,\chi^*}}{1+\sqrt{\rho'\,\chi^*_\mathrm{T}}} \tag{111}$$

The slip-layer thickness ρ' can be taken from Table 8 as a function of the material.

The relative stress gradient can be calculated using the equation (112) [11]:

$$\chi^* = \chi^*_\mathrm{P}(1+2q_\mathrm{s}) \tag{112}$$

with

$$\chi^*_\mathrm{P} = \frac{1}{5}$$

The value of χ_T^* for the standard reference test gear is obtained similarly by substituting $q_\mathrm{sT} = 2{,}5$ for q_s in equation (112).

Table 8 — Values for the slip layer thickness ρ'

Material[a]	ρ' [mm]
GG; $\sigma_\mathrm{B} = 150$ N/mm^2	0,312 4
GG, GGG (ferr.); $\sigma_\mathrm{B} = 300$ N/mm^2	0,309 5
NT, NV; for all hardness	0,100 5
St; $\sigma_\mathrm{S} = 300$ N/mm^2	0,083 3
St; $\sigma_\mathrm{S} = 400$ N/mm^2	0,044 5
V, GTS, GGG (perl. bai.); $\sigma_\mathrm{S} = 500$ N/mm^2	0,028 1
V, GTS, GGG (perl. bai.); $\sigma_\mathrm{S} = 600$ N/mm^2	0,019 4
V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 800$ N/mm^2	0,006 4
V, GTS, GGG (perl. bai.); $\sigma_{0,2} = 1000$ N/mm^2	0,001 4
Eh, IF(root); for all hardnesses	0,003 0
[a] See Table 2 for explanation of abbreviations used.	

7.6.3 $Y_{\delta\,\mathrm{rel\,T}}$ for static stress

$Y_{\delta\,\mathrm{rel\,T}}$ can be calculated using equations (113) to (117).

a) For steel with a well defined yield point, St[12]:

$$Y_{\delta\,\mathrm{rel\,T}} = \frac{1+0{,}93\ (Y_\mathrm{S}-1)\ \sqrt[4]{\dfrac{200}{\sigma_\mathrm{S}}}}{1+0{,}93\ \sqrt[4]{\dfrac{200}{\sigma_\mathrm{S}}}} \tag{113}$$

---

11) Applies for module $m = 5$ mm. The influence of size is covered by the factor Y_X (see 7.8).

12) See Table 2 for an explanation of the abbreviations used.

b) For steel with a steadily increasing elongation curve and 0,2 % proof stress, steel V and cast iron GGG (perl., bai.) [12]:

$$Y_{\delta\,\text{rel T}} = \frac{1+0{,}82\,(Y_\text{s}-1)\,\sqrt[4]{\dfrac{300}{\sigma_{0,2}}}}{1+0{,}82\,\sqrt[4]{\dfrac{300}{\sigma_{0,2}}}} \tag{114}$$

c) For steel Eh and IF(root) with stress up to crack initiation[12]:

$$Y_{\delta\,\text{rel T}} = 0{,}44\,Y_\text{S} + 0{,}12 \tag{115}$$

d) For steel NT and NV with stress up to crack initiation[12]:

$$Y_{\delta\,\text{rel T}} = 0{,}20\,Y_\text{S} + 0{,}60 \tag{116}$$

e) For cast iron GG and GGG (ferr.) with stress up to fracture limit[12]:

$$Y_{\delta\,\text{rel T}} = 1{,}0 \tag{117}$$

7.7 Relative surface factor, $Y_{\text{R rel T}}$

7.7.1 General

The surface factor, $Y_{\text{R rel T}}$, accounts for the influence on tooth-root stress of the surface condition in the tooth roots. Primarily, this is dependent on surface roughness in the tooth-root fillets.

The influence of surface condition on tooth-root bending strength does not depend solely on the surface roughness in the tooth-root fillets, but also on the size and shape (the problem of "notches within a notch"). This subject has not to date been sufficiently well studied for it to be taken into account in this International Standard. The method applied here is only valid when scratches or similar defects deeper than $2 \times Rz$ are not present.

NOTE $2 \times Rz$ is a preliminary estimated value.

In this International Standard, Method C of ISO 6336-3:1996 is used.

7.7.2 $Y_{\text{R rel T}}$ for reference and long life stresses

For all materials,

— if $Rz \leqslant 16$ µm then:

$$Y_{\text{R rel T}} = 1{,}0 \tag{118}$$

— If $Rz > 16$ µm then:

$$Y_{\text{R rel T}} = 0{,}9 \tag{119}$$

7.7.3 $Y_{\text{R rel T}}$ for static stress

For all materials, no dependence on root fillet roughness:

$$Y_{\text{R rel T}} = 1{,}0 \tag{120}$$

7.8 Size factor, Y_X

Y_X is used to allow for the influence of size on:

— the probable distribution of weak points in the material structure;

— the stress gradients, which in materials theory decrease with increasing dimensions;

— material quality;

— as regards quality of forging, presence of defects etc.

Method B of ISO 6336-3:1996 is used in this International Standard.

Y_X is calculated in accordance with Table 9.

Table 9 — Size factor (root), Y_X

Material [a]		Normal module m_n	Size factor Y_X
St, St (cast), V, GGG (perl., bai.), GTS (perl.)	for 3×10^6 to 10^{10} cycles	$m_n \leqslant 5$ $5 < m_n < 30$ $30 \leqslant m_n$	$Y_X = 1,0$ $Y_X = 1,03 - 0,006\ m_n$ $Y_X = 0,85$
Eh, IF (root), NT (nitr.), NV (nitr.), NV (nitrocar.)		$m_n \leqslant 5$ $5 < m_n < 25$ $25 \leqslant m_n$	$Y_X = 1,0$ $Y_X = 1,05 - 0,01\ m_n$ $Y_X = 0,8$
GG, GGG (ferr.)		$m_n \leqslant 5$ $5 < m_n < 25$ $25 \leqslant m_n$	$Y_X = 1,0$ $Y_X = 1,075 - 0,015\ m_n$ $Y_X = 0,7$
All materials	static	—	$Y_X = 1,0$

[a] See Table 2 for an explanation of the abbreviations used.

7.9 Minimum safety factor (tooth breakage), $S_{F\,min}$

For general aspects concerning safety factors, see clause 4; for calculation of the actual safety factor (tooth breakage), S_F, see 7.1.4. If not otherwise agreed between manufacturer and user, the following minimum safety factor (tooth breakage), $S_{F\,min}$, is applied in this International Standard:

$$S_{F\,min} = 1,2 \tag{121}$$

Annex A
(normative)

Special features of less common gear designs

A.1 Dynamic factor, K_V, for planetary gears

A.1.1 General

In gear trains which include multiple mesh gears such as idler gears and in epicyclic gearing, planet and sun gears, there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh.

Although values of K_V determined using the formulae in this International Standard shall be considered as unreliable, nevertheless they can be useful as preliminary assessments. It is recommended that, if possible, they should be re-assessed using a more accurate procedure.

Method A of ISO 6336-1:1996 should be preferred for the analysis of less common transmission designs. Refer to 6.1.1 of ISO 6336-1:1996 for further information.

A.1.2 Calculation of the equivalent mass of a gear pair with external teeth

Refer to 5.6.2

A.1.3 Resonance speed determination for less common gear designs

A.1.3.1 General

The resonance speed determination for less common gear designs should be made using Method A. However, other methods may be used to approximate the effects. Some examples are

a) pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter,

b) two rigidly connected coaxial gears,

c) one large wheel driven by two pinions,

d) planetary gears, and

e) idler gears.

A.1.3.2 Pinion shaft with diameter a mid-tooth, d_{m1}, about equal to the shaft diameter

The high torsional stiffness of the pinion shaft is to a great extent compensated by the shaft mass. Thus the resonance speed can be calculated in the normal way, using the mass of the pinion (toothed portion) and the normal mesh stiffness c_γ.

A.1.3.3 Two rigidly connected coaxial gears

The mass of the larger of the connected gears is to be included.

A.1.3.4 One large wheel driven by two pinions

As the mass of the wheel is normally much greater than the masses of the pinions, each mesh can be considered separately, i.e.:

a) as a pair comprising the first pinion and the wheel;

b) as a pair comprising the second pinion and the wheel.

A.1.3.5 Planetary gears

Because of the many transmission paths which include stiffnesses other than mesh stiffness, the vibratory behaviour of planetary gears is very complex. The calculation of dynamic load factors using simple formulae, such as Method B, is generally quite inaccurate. Nevertheless, Method B, modified as follows, can be used for a first estimate of K_V. This estimate should be verified be means of a subsequent detailed theoretical or experimental analysis, or on the basis of operating experience. See also the introductory comments to this annex.

a) Sun gear/planet gear

The equivalent mass for the determination of the resonance speed of the sun gear n_{E1} is given by

$$m_{red} = \frac{J^*_{pla}\, J^*_{sun}}{\left(pJ^*_{pla}\, r^2_{b\,sun}\right) + \left(J^*_{sun}\, r^2_{b\,pla}\right)} \tag{A.1}$$

where

J^*_{pla}, J^*_{sun} are the moments of inertia per unit facewidth of the sun gear and one planet gear respectively in kg mm²/mm;

$r_{b\,sun}$ $= 0{,}5\, d_{b\,sun}$;

$r_{b\,pla}$ $= 0{,}5\, d_{b\,pla}$;

p is the number of planet gears in the gear stage under consideration.

The value, m_{red}, determined from equation (A.1), shall be used in the equation for calculating N (see 5.6.2.2) where a mesh stiffness approximately equal to a single planetary gear shall be used for the mesh stiffness c_γ and the number of teeth on the sun gear shall be used for z_1.

Concerning planetary gears, it should be noted that F_t in equations (12), (13) and (14) (see 5.6.2.3) is equal to the total tangential load applied to the sun gear divided by the number of planet gears.

b) Planet gear/annulus gear rigidly connected to the gear case

In this case, the mass of the annulus gear can be assumed to be infinite. Thus, the equivalent mass becomes equal to the referred mass of the planet gear. This can be determined as follows:

$$m_{red} = \frac{J^*_{pla}}{r^2_{b\,pla}} \tag{A.2}$$

with the notation as above.

c) Planet gear/rotating annulus gear

In this case the referred mass of the annulus gear may be determined as for an external wheel and the planet gear equivalent mass calculated in accordance with equation (A.2). The procedure described in A.1.3.4 shall be used when the annulus gear meshes with several planet gears.

A.1.3.6 Idler gears

Approximate values can be obtained from the following when the driving and driven gears are roughly of the same size, with the idler gear also about the same size or a little larger:

— reduced mass

$$m_{\text{red}} = \frac{2}{\left(\frac{r_{\text{b1}}^2}{J_1^*} + \frac{2 r_{\text{b2}}^2}{J_2^*} + \frac{r_{\text{b3}}^2}{J_3^*} \right)} \quad \text{(A.3)}$$

— mesh stiffness

$$c_\gamma = 0{,}5(c_{\gamma 1,2} + c_{\gamma 2,3}) \quad \text{(A.4)}$$

where

J^*_1,J^*_2,J^*_3 are the moments of inertia per unit facewidth of the pinion, the idler and the wheel, respectively, in kilogram millimetres squared per millimetre (kg mm^2/mm);

$c_{\gamma\,1,2}$ is the mesh stiffness of the driver and idler gear pair;

$c_{\gamma\,2,3}$ is that of the idler and driven gear pair (see annex B for the determination of c_γ). More accurate analysis is recommended if the reference speed is in the range 0,6 < < 1,5.

If the idler is substantially larger than the driving and driven gears or, if the driving gear or driven gear is substantially smaller than the two other gears, K_v can be calculated separately for each meshing pair, i.e.

— for the driver-idler gear combination, and

— for the idler-driven gear combination.

Values of m_{red} calculated in accordance with the above may be substituted in equation (7) to determine the resonance speed.

An accurate analysis is recommended for cases not mentioned here.

Annex B
(normative)

Tooth stiffness parameters c' and c_γ

B.1 General

A tooth stiffness parameter represents the requisite load over a 1 mm facewidth, directed along the line of action[13] to produce in line with the load, the deformation amounting to 1 µm, of one or more pairs of deviation-free teeth in contact.

Single stiffness c' is the maximum stiffness of a single tooth pair of a spur gear pair. It is approximately equal to the maximum stiffness of a tooth pair in single pair contact[14]. c' for helical gears is the maximum stiffness normal to the helix of one tooth pair.

Mesh stiffness c_γ is the mean value of stiffness of all the teeth in a mesh.

Method B from ISO 6336-1:1996, used in this International Standard, is applicable in the ranges $x_1 \geqslant x_2$ and $-0,5 \geqslant (x_1 + x_2) \leqslant 2$.

B.2 Single stiffness c'

B.2.1 Calculation of c'

For specific loading $F_t K_A / b \geqslant 100$ N/mm²:

$$c' = 0,8\, c'_{th}\, C_R\, C_B \cos \beta \tag{B.1}$$

B.2.2 Theoretical single stiffness, c_{th}'

$$c'_{th} = \frac{1}{q'} \tag{B.2}$$

where

$$q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + (C_4 x_1) + \frac{(C_5 x_1)}{z_{n1}} + (C_6\, x_2) + \frac{(C_7 x_2)}{z_{n2}} + \left(C_8 x_1^2\right) + \left(C_9 x_2^2\right) \tag{B.3}$$

Table B.1 — Constants for equation (B.3)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0,047 23	0,155 51	0,257 91	– 0,006 35	– 0,116 54	– 0,001 93	– 0,241 88	0,005 29	0,001 82

13) The tooth deflection can be determined approximately using F_t (F_m F_{tH}) instead of F_{bt}. Conversion from F_t to F_{bt} (load tangent to the base cylinder) is covered by the relevant factors, or the modifications resulting from this conversion can be ignored when compared with other uncertainties (e.g. tolerances on the measured values).

14) c' at the outer limit of single pair tooth contact can be assumed to approximate the maximum value of single stiffness when $\varepsilon_\alpha > 1,2$.

B.2.3 Gear blank factor, C_R

C_R = 1 for gears made from solid disc blanks.

For other gears:

$$C_R = 1 + \frac{\ln(b_s / b)}{5e^{s_R/(5m_n)}} \tag{B.4}$$

Boundary conditions:

when $b_s/b < 0{,}2$ substitute $b_s/b = 0{,}2$;

when $b_s/b > 1{,}2$ substitute $b_s/b = 1{,}2$.

See Figure B.1 for symbols.

B.2.4 Basic rack factor, C_B

C_B can be obtained from equation (B.5)

$$C_B = \left[1 + 0{,}5\left(1{,}2 - \frac{h_{fP}}{m_n}\right)\right]\left[1 - 0{,}02(20° - \alpha_{Pn})\right] \tag{B.5}$$

B.2.5 Additional information

a) Internal gearing: approximate values of the theoretical single stiffnesses of internal gear teeth can be determined from equations (B.2), (B.3), by the substitution of infinity for z_{n2}.

b) Specific loading $(F_t K_A)/b < 100$ N/mm:

$$c' = 0{,}8\, c'_{th}\, C_R\, C_B \cos\beta \left[\frac{F_t K_A}{100\, b}\right]^{0{,}25} \tag{B.6}$$

c) The above is based on steel gear pairs. For other materials and material combinations, refer to ISO 6336 1:1996, clause 9.

B.2.6 Mesh stiffness, c_γ

For spur gears with $\varepsilon_\alpha \geqslant 1{,}2$ and helical gears with $\beta \leqslant 30°$, the mesh stiffness:

$$c_\gamma = c'(0{,}75\,\varepsilon_\alpha + 0{,}25) \tag{B.7}$$

with c' according to equation (B.1).

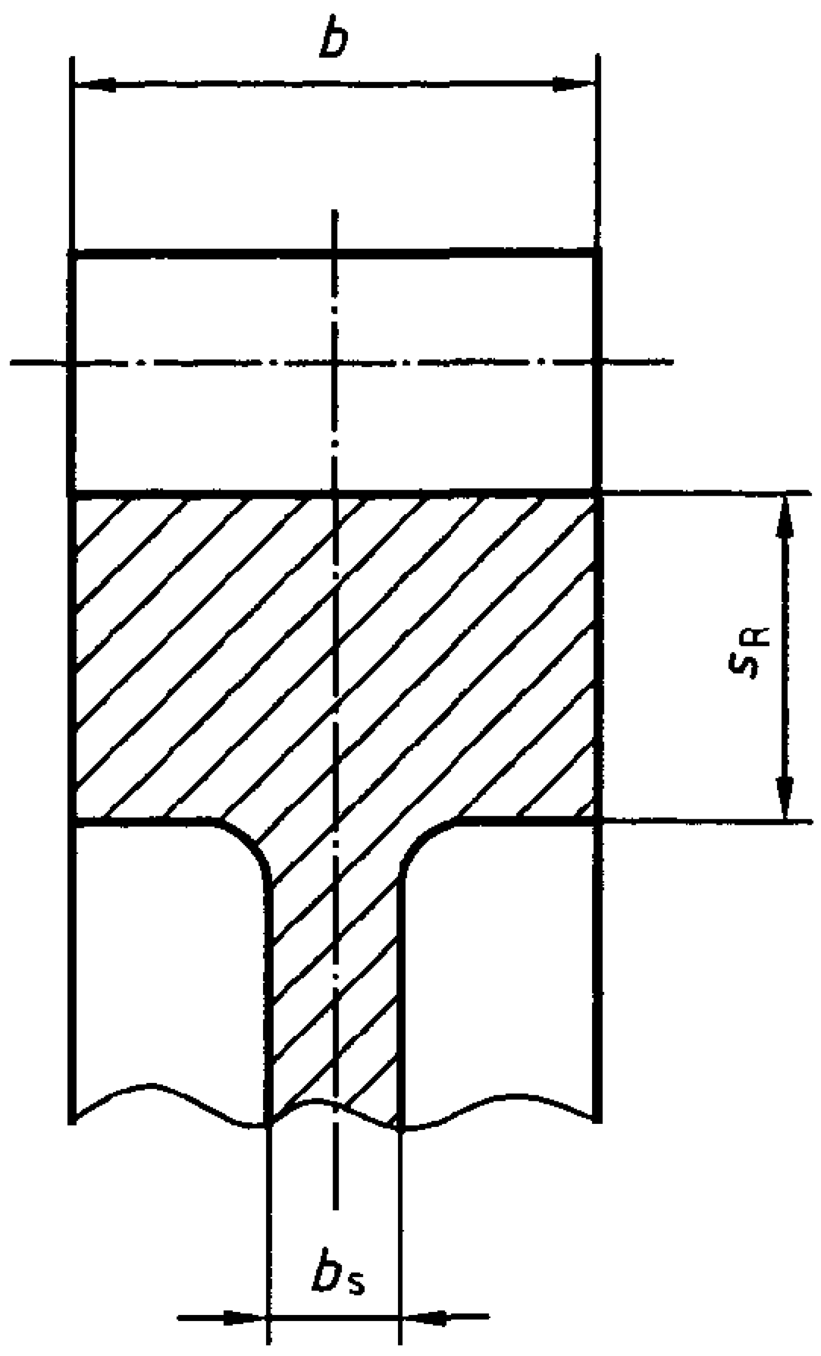


Figure B.1 — Symbols for determination of C_R

Annex C
(informative)

Guide values for application factor, K_A

C.1 Establishment of application factors

Application factors can best be established from a thorough analysis of service experience with a particular application (see ISO TR 10495). When service experience is lacking, a thorough analytical investigation should be made.

The factor K_A is used to modify the value F_t, to take into account loads additional to nominal loads which are imposed on the gears from external sources. If it is not possible to determine the equivalent tangential load (see 5.2) by comprehensive system analysis or from measured values using a suitable cumulative damage criterion, the empirical guidance values in Table C.1 may be used.

C.2 Approximate values for the application factors

Table C.1 provides typical values for application factors which may be used if service experience is lacking or when a detailed analysis is not available. The table should be used with caution since much higher values have occurred in some applications. Values as high as 10 have been used.

The values only apply to transmissions which operate outside the resonance speed range under relatively steady loading. If operating conditions involve unusually heavy loading, motors with high starting torques, intermittent service or heavy repeated shock loading, the safety of the static and limited-life load capacity of the gears shall be verified (see ISO 6336-1, ISO 6336-2 and ISO 6336-3).

Table C.1 — Application factors, K_A

Working characteristics of the driving machine	**Working characteristics of the driven machine**			
	Uniform	Light shocks	Moderate shocks	Heavy shocks
	Application factors, K_A			
Uniform	1,00	1,25	1,50	1,75
Light shocks	1,10	1,35	1,60	1,85
Moderate shocks	1,25	1,50	1,75	2,00
Heavy shocks	1,50	1,75	2,00	2,25 or higher

EXAMPLES

a) Turbine/generator

In this system, short-circuit torques of up to six times the nominal torque can occur. Such overloads can be shed by means of safety couplings.

b) Electric motor/compressor

If pump frequency and torsional natural frequency coincide, considerable alternating stresses can occur.

c) Heavy plate and billet rolling mills

Initial pass-shock-torques up to six times the rolling torque shall be taken into account in these cases.

d) Drives with synchronous motors

Alternating torques up to five times the nominal torque can occur briefly (approximately 10 amplitudes) on starting; however, hazardous alternating torques can often be completely avoided by the appropriate detuning measures.

Information and numerical values provided here cannot be generally applied. The magnitude of the peak torque depends on the mass spring system, the forcing term, safety precautions (safety coupling, protection for unsynchronised switching of electrical machines) etc.

Thus in critical cases careful analysis is essential. It is then recommended that agreement be reached on suitable actions.

Application factors stated in the purchase order should be taken into consideration as minimum required values. Also see clause 4.

Where there are additional inertial masses, torques resulting from the flywheel effect are to be taken into consideration. Occasionally, braking torque provides the maximum loading and thus influences calculation of load capacity.

It is assumed the gear materials used should have adequate overload capacity. When materials used have only marginal overload capacity, designs should be laid out for endurance at peak loading.

The K_A value for moderate, average and heavy shocks can be reduced by using hydraulic couplings or torque matched elastic coupling, and especially vibration attenuating couplings when the characteristics of the couplings so permit.

Table C.2 — Examples for driving machines with various working characteristics

Working characteristics	Driving machine
Uniform	Electric motor (e.g. d.c. motor), steam or gas turbine with uniform operation[a] and small rarely occurring starting torques[b]
Light shocks	Steam turbine gas turbine, hydraulic or electric motor (large, frequently occurring starting torques[b])
Moderate shocks	Multiple cylinder internal combustion engines
Heavy shocks	Single cylinder internal combustion engines

[a] Based on vibration tests or on experience gained from similar installations.

[b] See service life graph Z_{NT} Y_{NT} for the material in ISO 6336-2 and ISO 6336-3. For consideration of momentarily acting overload torques, see examples following Table C.1.

Table C.3 — Examples of working characteristics of driven machines

Working characteristics	Driven machines
Uniform	Steady load current generator; uniformly loaded conveyor belt or platform conveyor; worm conveyor; light lifts; packing machinery; feed drives for machine tools; ventilators; lightweight centrifuges; centrifugal pumps; agitators and mixers for light liquids or uniform density materials; shears; presses, stamping machines[a]; vertical gear, running gear[b].
Light shocks	Non-uniformly (i.e. with piece or batched components) loaded conveyor belts or platform conveyors; machine tool main drives; heavy lifts; crane slewing gear; industrial and mine ventilator; heavy centrifuges; centrifugal pumps; agitators and mixers for viscous liquids or substances of non-uniform density, multi-cylinder piston pumps, distribution pumps; extruders (general); calendars; rotating kilns; rolling mill stands[c] (continuous zinc and aluminium strip mills, wire and bar mills).
Moderate shocks	Rubber extruders; continuously operating mixers for rubber and plastics; ball mills (light); wood-working machine (gang saws, lathes); billet rolling mills[c, d]; lifting gear; single cylinder piston pumps.
Heavy shocks	Excavators (bucket wheel drives), bucket chain drives; sieve drives; power shovels, ball mills (heavy); rubber kneaders; crushers (stone, ore); foundry machines; heavy distribution pumps; rotary drills; brick presses; debarking mills; peeling machines; cold strip[c, e]; briquette presses; breaker mills.

a Nominal torque = maximum cutting, pressing or stamping torque.

b Nominal torque = maximum starting torque.

c Nominal torque = maximum rolling torque.

d Torque from current limitation.

e K_A up to 2,0 because of frequent strip cracking.

Annex D
(informative)

Guide values for crowning and end relief of teeth of cylindrical gears

D.1 General

Well-designed crowning and end relief have a beneficial influence on the distribution of load over the facewidth of a gear (see 5.7). Design details should be based on a careful estimate of the deformations and manufacturing deviations of the gearing of interest. If deformations are considerable, helix angle modification may be superposed over crowning or end relief, but well designed helix modification is preferable.

D.2 Amount of crowning C_β

The following non-mandatory rule is drawn from experience; the amount of crowning (see Figure D.1) necessary to obtain acceptable distribution of load can be determined as follows.

Subject to the limitations 10 µm $\leqslant C_\beta \leqslant$ 40 µm plus a manufacturing tolerance of 5 µm to 10 µm, and that the value b_{cal}/b would have been greater than 1 had the gears not been crowned, $C_\beta \approx 0{,}5\, F_{\beta x\,cv}$.

To avoid excessive loading of tooth ends, the crowning amount shall be calculated as:

$$C_\beta = 0{,}5\,(f_{sh} + f_{H\beta}) \tag{D.1}$$

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate for deformation at mid-facewidth, the following value may be substituted:

$$C_\beta = 0{,}5 f_{H\beta} \tag{D.2}$$

Subject to the restriction 10 µm $\leqslant C_\beta \leqslant$ 25 µm plus a manufacturing tolerance of about 5 µm, 60 % to 70 % of the above values are adequate for extremely accurate and reliable high speed gears.

See Figure D.1.

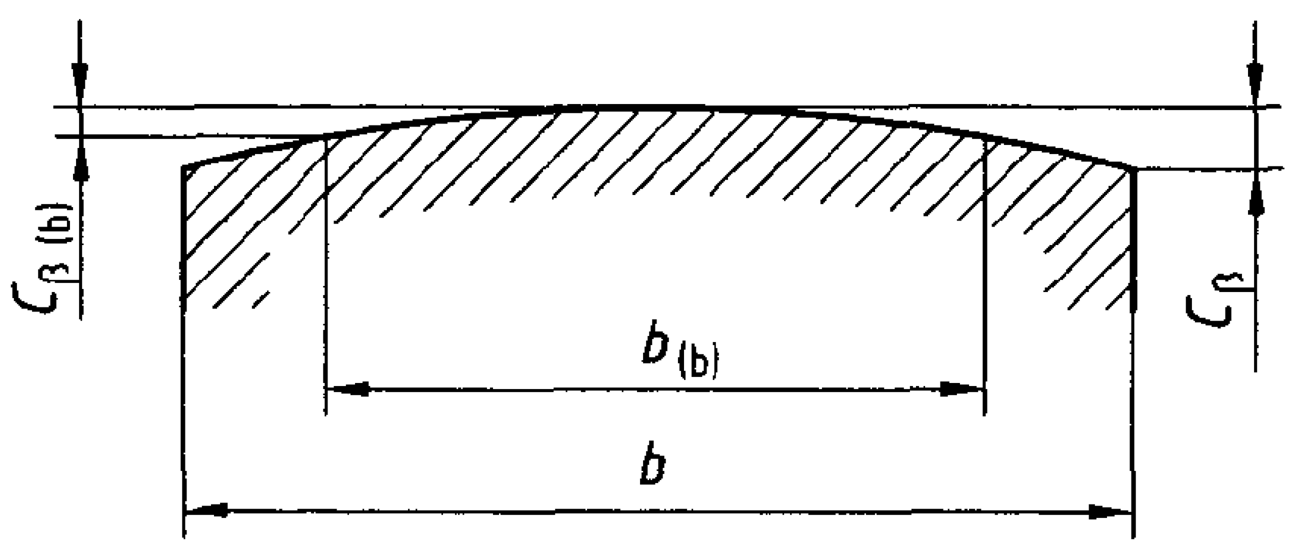


Figure D.1 — Amount of crowning $C_{\beta\,(b)}$ and width $b_{(b)}$

D.3 Amount $C_{\mathrm{I(II)}}$ and width $b_{\mathrm{I(II)}}$ of end relief

D.3.1 Method C.1

This method is based on an assumed value for the equivalent misalignment of the gear pair, without end relief and on the recommendations for the amount of gear crowning.

a) Amount of end relief (see Figure D.2)

For through hardened gears: $C_{\mathrm{I(II)}} \approx F_{\beta\mathrm{x\,cv}}$ plus a manufacturing tolerance of 5 to 10 µm.

Thus, by analogy with $F_{\beta\mathrm{x\,cv}}$ in D.2, $C_{\mathrm{I(II)}}$ should be approximately

$$C_{\mathrm{I(II)}} = f_{\mathrm{sh}} + 1{,}5\, f_{\mathrm{H}\beta} \qquad \text{(D.3)}$$

For surface hardened and nitrided gears: $C_{\mathrm{I(II)}} \approx 0{,}5\, F_{\beta\mathrm{x\,cv}}$ plus a manufacturing tolerance of 5 to 10 µm.

Thus, by analogy with $F_{\beta\mathrm{x\,cv}}$ in D.2, $C_{\mathrm{I(II)}}$ should be approximately

$$C_{\mathrm{I(II)}} = 0{,}5\left(f_{\mathrm{sh}} + 1{,}5\, f_{\mathrm{H}\beta}\right) + 1{,}5\, f_{\mathrm{H}\beta} \qquad \text{(D.4)}$$

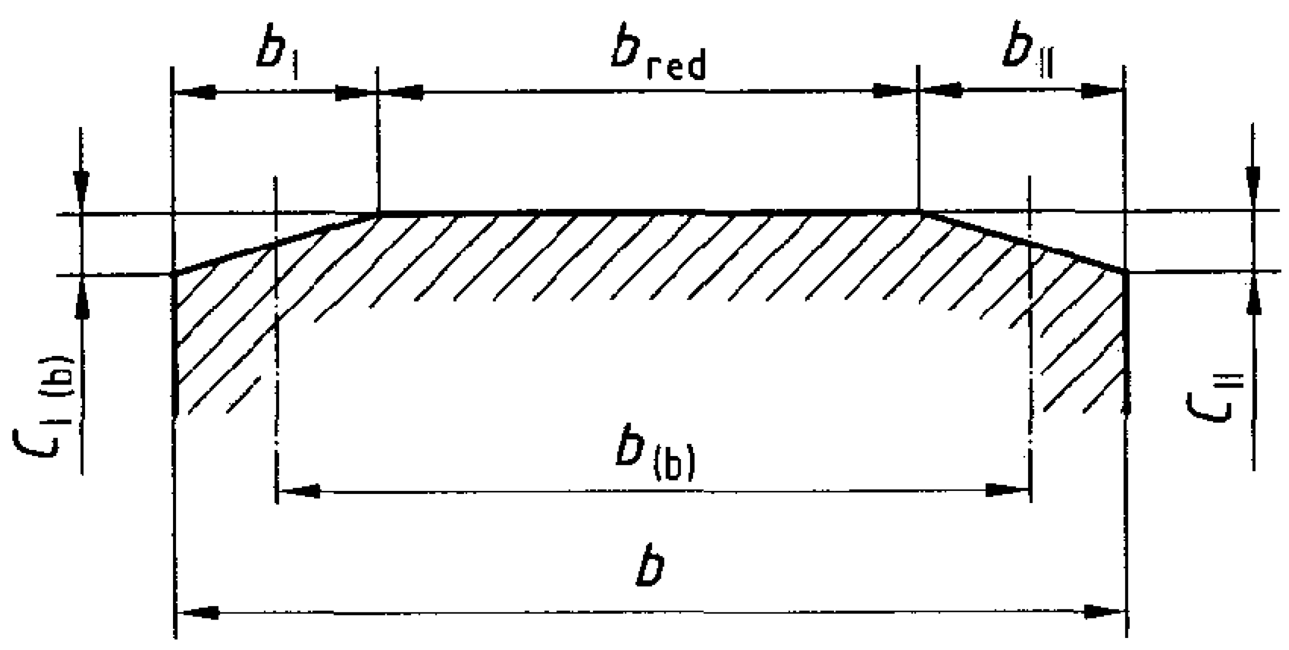


Figure D.2 — Amount $C_{\mathrm{I(II)(b)}}$ and width $b_{\mathrm{(b)}}$ of end relief

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate deformation, proceed in accordance with equation (D.2).

60 % to 70 % of the above values are appropriate for very accurate and reliable gears with high tangential velocities.

b) Width of end relief

For approximately constant loading and higher tangential velocities: $b_{\mathrm{I(II)}}$ is the smaller of the values (0,1 b) or (1,0 m)

The following is appropriate for variable loading, low and average speeds:

$$b_{\mathrm{red}} = (0{,}5 \text{ to } 0{,}7)\, b \qquad \text{(D.5)}$$

D.3.2 Method C.2

This method is based on the deflection of gear pairs assuming uniform distribution of load over the facewidth:

$$\delta_{\mathrm{bth}} = F_{\mathrm{m}} / (b\, c_{\gamma}) \qquad \text{(D.6)}$$

where $F_m = F_t K_A K_v$

For highly accurate and reliable gears with high tangential velocities, the following are appropriate:

$$C_{I(II)} = (2 \text{ to } 3)\delta_{bth} \tag{D.7}$$

$$b_{red} = (0{,}8 \text{ to } 0{,}9)b \tag{D.8}$$

For similar gears of lesser accuracy:

$$C_{I(II)} = (3 \text{ to } 4)\delta_{bth} \tag{D.9}$$

$$b_{red} = (0{,}7 \text{ to } 0{,}8)b \tag{D.10}$$

Bibliography

[1] ISO 701:1998, *International gear notation — Symbols for geometrical data*

[2] ISO 4288:1998, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Rules and procedures for the assessment of surface texture*

[3] ISO 9083:2001, *Calculation of load capacity of spur and helical gears — Application to marine gears*

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